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Calculation of Tabarru Funds using Makeham's Mortalita Law and Gompertz's Mortalita Law using the Cost of Insurance Method

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Abstracts

Tabarru funds have the meaning of a collection of funds given by insurance participants as a virtue fund with sincere intentions for the purpose of helping one participant with another if one of them gets a disaster. The tabarru fund management mechanism in Indonesia uses two types of operational systems, namely the product saving system (savings) and the non-saving product system. Management with a saving product system uses a savings mechanism with 5% for management funds. Meanwhile, in the non-saving product system with a no-savings mechanism, the amount and its management are not yet known, which will cause confusion for the community in the calculation. The cost of insurance method is one method that can be used to calculate tabarru funds. This method calculates tabarru funds by multiplying the percentage of tabarru funds by the cost of coverage. The percentage of tabarru funds is searched through the mortality table, management fee, and investment level. From the management fee of 25% and the investment rate of 5%, the percentage of tabarru funds using Makeham's mortality law is $COI_0 = 0,46171988$ for men and $COI_0 = 1,199806538$ for men. Meanwhile, Gompertz's law of mortality obtained $COI_0 = 0,297362966$ for men and for women $COI_0 = 1,779070406$.

Keywords: time minimization, distribution of ballots, greedy algorithms

1. Introduction

Insurance is an agreement made by two parties where the first party is the insured and the second party is the insurer [1]. The first party or the insured has the obligation to pay premiums to the second party and the second party or the insurer has the responsibility to guarantee for the losses suffered by the first party or the insured in accordance with the agreement (contract) that has been agreed [2].

There are two types of life insurance that are developing in society, namely conventional life insurance and sharia life insurance. In concept, all losses experienced by insurance participants in conventional life insurance are borne by the insurance company [3], while in sharia life insurance losses are shared through tabarru funds. In sharia life insurance, tabarru funds themselves have the meaning of a collection of funds given by insurance participants as a virtue fund with sincere intentions for the purpose of helping one participant with another if one of them gets a disaster. From this understanding, participants voluntarily submit tabarru funds to sharia life insurance as operators in administrative and investment management [4].

The tabarru fund management mechanism in Indonesia uses two types of operational systems, namely the product saving system (savings) and the non-saving product system . Management with a saving product system uses a savings mechanism with 5% for management funds [5]. Meanwhile, in the non-saving product system with a no-savings mechanism, the amount and management are not yet known [6]. Determining the amount of tabarru funds in sharia life insurance requires an analysis of various factors [7]. One of the factors that can be analyzed is mortality (rate of death)[8]. The mortality table is a tool used to calculate the probability of a person's life and death in a certain period of time. "The mortality table will contain the probability of a person dying based on their age in the group of people who are insured (policy holders)"[9]. This table will describe as closely as possible the true odds of the insured group of people. The death rate of a group or population can be analyzed using the law of mortality by determining the survival function, survival probability, and mortality probability through its distribution function. This research uses Makeham's law of mortality and Gompertz's law of mortality. The two laws of mortality are distributions commonly used to describe a person's age of death [10].

Gompertz's law of mortality and Makeham's law of mortality have similarities in calculating the mortality rate of a population, but they also have differences that affect the level of accuracy of actuarial functions, namely in the parameters that must be estimated. In Gompertz's law of mortality there are two parameters that need to be estimated while in Makeham's law of mortality there are three parameters that must be estimated [11]. In Gompertz's Mortalita law, the estimated parameters are the chance of life and the chance of death, while Makeham's Mortalita law estimates the chance of life, chance of death, and other opportunities related to a person's age such as the chance of an accident, the chance of getting sick, and the chance of being hit by natural disasters that can occur. happen anytime [12]. In addition to comparing the effectiveness of the two mortality laws, this study will also focus on the mechanism for calculating tabarru funds without an element of savings. In this study, the Cost of insurance (COI) method was used in calculating tabarru funds. Based on this explanation, the author conducted a research entitled "Calculation of Tabarru Funds Using Makeham's Mortalita Law and Gompertz Mortalita Law Using the Cost Of Insurance Method".

2. Experiments Procedure

This study uses a case study of tabarru fund payments with data taken from the 2011 Indonesian Mortality Table (TMII) for male and female sampling. Then it will be discussed or studied using Makeham's mortality law and Gompertz's mortality law, and the amount of tabarru funds will be calculated using the Cost of insurance (COI) method. This study uses the following steps [13].

1. Calculating the estimated parameter values for Makeham's mortality law and Gompertz's mortality law in the Indonesian Mortality Table (TMI) in 2011 for male and female data samples.
2. Calculating the probability of living and the probability of dying based on Makeham's law of mortality and Gompertz's law of mortality.
3. Assuming the amount of management costs and the level of investment.
4. Calculating the percentage of tabarru funds using the Cost of Insurance (COI) method based on Makeham's mortality law and Gompertz's mortality law.

3. Result and Discussion

Estimated parameters of Gompertz and Makeham parameters

Gompertz and Makeham parameter estimates can be determined using a simple regression equation as follows, [12]

$$\ln\left[\ln\left(\frac{1}{1-q_x}\right)\right] = x\ln C + \ln\left[\frac{AB}{\ln C}(C-1)\right] \quad (1)$$

From the regression equation (1), the regression form is obtained as follows:

$$y = \alpha x + \beta \quad (2)$$

where

$$y = \ln\left(\ln\frac{1}{1-q_x}\right)$$

$$\alpha = \ln C$$

$$\beta = \ln\left[\frac{AB}{\ln C}(C-1)\right]$$

So that the resulting simple regression estimator is as follows

$$\hat{y} = ax_i - b$$

with a, b are α and β estimators. The values of a and b can be calculated using the *Ordinary Least Square* (OLS) method which is minimizing the number of squares of error, we get

$$R = \sum_{i=1}^{N}\varepsilon_i^2 = \sum_{i=1}^{N}\left(y_i - ax_i - b\right)^2 \quad (3)$$

where x is the age and y_i is the model estimator. Parameters of B and C in Makeham's law of mortality and Gompertz's law of mortality have the same value [14]. Meanwhile, in Gompertz's law of mortality, it is a special case of Makeham's law of mortality, which makes the magnitude of parameter A, which is 0[15]. To find the parameters A, B, and C of the mortality law, it is as follows: [12]

1. $\frac{dR}{da} = 0$ will be obtained for the estimate $a = \frac{\sum x_i y_i N - \sum x_i \sum y_i}{\sum x_i^2 N - \left(\sum x_i\right)^2}$. Estimation a is an estimator of the parameters of so that the parameter C for men and women is 1.076115594 and 1.060361630.
2. $\frac{dR}{db} = 0$ will be obtained for the estimate Estimation b is an estimator of the parameters of so that the parameter B is 0.257133136 for men and 2,639400647 for women.
3. For parameter A of 0.185135483 for men and 0.179374078 for women in Makeham's mortality law.

Establishment of a New Mortality Table

Creating the Makeham mortality table, the following formulas were used:

1. Determine the life expectancy of a person aged x year
$$p_x = \exp\left[-\left(A + \frac{BC^x}{\ln C}(C-1)\right)\right] \quad (4)$$
2. Calculate the probability of death of a person aged x year
$$q_x = 1 - \exp\left[-\left(A + \frac{BC^x}{\ln C}(C-1)\right)\right] \quad (5)$$

Below is the Makeham mortality table for male and female $\left(p_x\right)$

Table 1. Makeham mortality table for men and women $\left(p_x\right)$

x	Laki-laki	Perempuan
0	0,636395595	0,055152352
1	0,62360227	0,046806377
2	0,610122317	0,039331881
3	0,595941648	0,032705701
4	0,581049569	0,026894653
5	0,565439412	0,021856483
...		
110	0	0
111	1	1

Furthermore, the Gompertz mortality table can be determined using the following formulas

1. Determine the life expectancy of a person aged x year

$$p_x = \exp\left(\frac{BC^x}{\ln C}(C-1)\right) \quad (6)$$

2. Calculate the probability of death of a person aged x year

$$q_x = 1-\exp\left(\frac{BC^x}{\ln C}(C-1)\right) \quad (7)$$

Below is the Gompertz mortality table for men and women (p_x)

Table 2. Gompertz mortality table for men and women (p_x)

x	Laki-laki	Perempuan
0	0,765826664	0,065988037
1	0,750431415	0,056002344
2	0,734209889	0,047059346
3	0,717145135	0,039131332
4	0,699224282	0,032178598
5	0,680439309	0,026150587
...		
110	0	0
111	1	1

Calculation of the Percentage of Tabarru Funds Using the Cost Of Insurance Method

The calculation of tabarru funds using the cost of insurance method is carried out using several assumptions about the level of investment and management costs to calculate it [15]. The investment level assumptions (i) used are 5%, 10%, and 20%. To perform a simulation of the calculation of tabarru funds using the cost insurance method, it can be done as follows:

$$COI_x = \frac{vq_x}{1-a} \quad (8)$$

with v = factor's value, q_x = probability that a person dies at age x and a is a cost of management. Assuming the management fee is 25%, the following table is the calculation of tabarru funds for male using the Gompertz method.

Table 2. Gompertz method of calculating male tabarru funds for 25% management fees

x	COI_x		
	$i = 5\%$	$i = 10\%$	$i = 20\%$
0	0,297362966	0,283846468	0,260192595
1	0,316912489	0,302507376	0,277298428
2	0,337511252	0,322169832	0,295322346
3	0,359180781	0,342854382	0,314283183
...			
110	1,26984127	1,212121212	1,111111111
111	1,26984127	1,212121212	1,111111111

Furthermore, the possibility of tabarru funds for women assuming a management fee of 25% can be seen in Table 3.

Table 3. Gompertz method of calculating female tabarru funds for 25% management fees

x	COI_x		
	$i = 5\%$	$i = 10\%$	$i = 20\%$
0	1,186046938	1,132135713	1,03779107
1	1,198727183	1,144239583	1,048886285
2	1,21008337	1,15507958	1,058822949
3	1,22015069	1,164689295	1,067631853
...			
110	1,904761905	1,818181818	1,666666667
111	1,904761905	1,818181818	1,666666667

Using equations (7) and (8) it can also be calculated the amount of tabarru funds for male and female using the Makeham method. Assuming a management fee of 25%, the following are the results of the calculation of tabarru funds for male.

Table 4. Makeham method of calculating male tabarru funds for 25% management fees

x	COI_x		
	$i = 5\%$	$i = 10\%$	$i = 20\%$
0	0,46171988	0,440732612	0,404004895
1	0,477965371	0,456239672	0,4182197
2	0,495082772	0,47257901	0,433197426
3	0,51308997	0,489767699	0,448953724
...			
110	1,26984127	1,212121212	1,111111111
111	1,26984127	1,212121212	1,111111111

Furthermore, the following table states the possibility of tabarru funds with Makeham's method for women assuming a management fee of 25% can be seen in Table 5.

Table 5. Makeham method of calculating female tabarru funds for 25% management fees

x	COI_x		
	$i = 5\%$	$i = 10\%$	$i = 20\%$
0	1,199806538	1,145269877	1,04983072
1	1,2104046	1,155386209	1,059104025
2	1,219896024	1,164446205	1,067409021
3	1,228310221	1,172477938	1,074771444
...			
110	1,904761905	1,818181818	1,666666667
111	1,904761905	1,818181818	1,666666667

4. Conclusion

The calculation of tabarru funds by applying Gompertz and Makeham's mortality law on the Cost Of Insurance (COI) method has a difference because there is parameter A which states factors that take into account deaths other than age. In addition, the factors that influence the calculation of tabarru funds using the COI method include gender, the level of investment and the amount of management costs. For example, the higher the investment, the lower the percentage of tabarru funds.

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