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Numerical computational approach for 6th order boundary value problems

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ABSTRACT
This study introduces numerical computational methods that employ fourth-kind Chebyshev polynomials as basis functions to solve sixth-order boundary value problems. The approach transforms Boundary Value Problems (BVPs) into a set of linear algebraic equations expressed as unidentified Chebyshev coefficients. These coefficients are subsequently resolved using matrix inversion. Numerical simulations are conducted to verify the appropriateness and effectiveness of this method, demonstrating its simplicity and superior performance compared to existing solutions. Furthermore, a graphical representation of the method's solution is incorporated.

Keywords:
Approximate solution; Boundary value problems; Collocation; Fourth kind Chebyshev polynomials;

Introduction

Boundary Value Problems (BVPs) arise when a set of ordinary differential equations has solution values and derivatives specified at certain points. Specifically, a two-point BVP involves determining the solution and derivatives at the boundaries. BVPs play a crucial role in mathematically simulating various real-world phenomena, including viscoelastic flow, heat transfer, and engineering sciences. To address BVPs, numerous numerical methods have been developed and explored. Several notable approaches have been investigated to solve BVPs. These strategies encompass utilizing global phase integral techniques to estimate eigenvalues in sixth-order BVPs [1]. Additionally, a comparison was conducted between B-spline interpolation and finite difference, finite element, and finite volume methods for two-point BVP [2]. Other methodologies involve employing homotopy perturbation methods to address sixth-order BVPs [3] and utilizing non-polynomial splines to solve sixth-order BVPs [4]. Furthermore, there is innovation introduced through a novel cubic B-spline method for linear fifth-order BVP [5].

Other techniques include applying the collocation method to solve sixth-order BVPs [6], utilizing the Daftardar Jafari method for numerical solutions of fifth and sixth-order nonlinear BVPs [7], and employing interpolation subdivision schemes for the numerical solution of two-point BVPs [8, 9]. Additionally, there is the development of a subdivision scheme-based collocation algorithm for fourth-order BVP [10]. Further methods involve using He polynomials in variational iteration methods to solve seventh-order BVPs [11] and applying power series approximation methods for the numerical solution of nth-order BVPs [12]. Another approach includes utilizing the tau collocation approximation method to solve first and second-order ordinary differential equations [13]. Overall, this review is dedicated to the numerical solutions of sixth-order BVPs and illustrates the various approaches that have been explored in the literature:

$$v^{vi}(t) + \mu_1(t)v^{v}(t) + \mu_2(t)v^{iv}(t) + \mu_3(t)v^{iii}(t) + \mu_4(t)v^{ii}(t) + \mu_5(t)v^{i}(t) + \mu_6(t)v(t) = g(t)\ , t \in [a,b] \tag{1}$$

with boundary conditions

$$v^{i}(a) = \alpha_i\ , v^{i}(b) = \beta_i, i = 0,1,2, \tag{2}$$

Where α_0 , α_1 , α_2 and β_0 , β_1, β_2 are given real constants, $\mu_i(t),\ i = 0,1,2,\cdots n$ and $g(t)$ are known functions on the an interval $\in [a,b]$ and $v(t)$ is the unknown function to be determined

Basic Definition

1. Chebyshev polynomials of the fourth kind

Chebyshev polynomials of the fourth type are orthogonal polynomials related to weight functions $(x) = \sqrt{\frac{1-t}{1+t}}\ \forall\ t \in [-1,1]$. The Chebyshev polynomials of the fourth kind are defined by $W_n(t) = \frac{\sin\left(n+\frac{1}{2}\right)\theta}{\sin\left(\frac{\theta}{2}\right)}$ with $W_0(t) = 1$ and $W_1(t) = 2t + 1$.

Hence, the first few Chebyshev Polynomials of the fourth kind are given below:

$Q_0(t) = 1, Q_1(x) = 2t + 1, Q_2(t) = 4t^2 + 2t - 1, Q_3(t) = 8t^3 + 4t^2 - 3t - 1,$

2. Shifted Chebyshev polynomials of the fourth kind

The fourth kind of Shifted Chebyshev Polynomials serves as orthogonal polynomials with respect to a specific weight function.

$W^*(t) = \sqrt{\frac{1-t}{t}}\ \forall\ t \in [0,1]$ with starting values ${Q^*}_0(t) = 1$ and ${Q^*}_1(t) = 4t - 1$.

Hence, the first few Shifted Chebyshev Polynomials of the fourth kind are given below:

${Q^*}_0(t) = 1, {Q^*}_1(t) = 4t - 1, {Q^*}_2(t) = 16t^2 - 12t + 1, {Q^*}_3(t) = 64t^3 - 80t^2 + 24t - 1$

3. Absolute Error

We defined absolute error as follows in this study: Absolute Error=$|\mathrm{V}(t) - v(t)|;\ 0 \le t \le 1$, where $\mathrm{V}(t)$ is the exact solution and $v(t)$ is the approximate solution

Methods

The study employed the fourth-kind Chebyshev polynomials as an approximation method, utilizing the following form:

$$v(t) = \sum_{i=0}^{n} Q(t)\, a_i \tag{3}$$

The unknown constants to be determined are $a_i\ , i = 0(1)n$

Thus, by differentiating equation (3) for nth-times as functions of t and substituting resulting solution into question (1), we have

$$\sum_{i=0}^{n} Q^{vi}(t)\,a_i + \mu_1(t)\sum_{i=0}^{n} Q^{v}(t)\,a_i + \mu_2(t)\sum_{i=0}^{n} Q^{iv}(t)\,a_i + \mu_3(t)\sum_{i=0}^{n} Q^{iii}(t)\,a_i + \mu_4(t)\sum_{i=0}^{n} Q^{ii}(t)\,a_i + \mu_5(t)\sum_{i=0}^{n} Q^{i}(t)\,a_i + \mu_6(t)\sum_{i=0}^{n} Q(t)\,a_i = g(t) \quad (4)$$

Let $\eta(t) = \sum_{i=0}^{n} Q^{vi}(t)\,a_i$, $\tau(t) = \sum_{i=0}^{n} Q^{v}(t)\,a_i$, $\varsigma(t) = \sum_{i=0}^{n} Q^{iv}(t)\,a_i$, $\xi(t) = \sum_{i=0}^{n} Q^{iii}(t)\,a_i$, $\gamma(t) = \sum_{i=0}^{n} Q^{ii}(t)\,a_i$, $\chi(t) = \sum_{i=0}^{n} Q^{i}(t)\,a_i$, $\omega(t) = \sum_{i=0}^{n} Q(t)\,a_i$

The system of linear algebraic equations involving (n+1) unknown constants $a_i s$ is derived by collocating equation (4) at evenly spaced points $t_i = a + \frac{(b-a)i}{n}$, $(i = 0(1)n)$. Additional equations are derived from Eq. (2) and are expressed in matrix form:

$$\begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} & & W_{1n} \\ W_{21} & W_{22} & W_{23} & W_{24} & \cdots & W_{2n} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ W_{m1} & W_{m2} & W_{m3} & W_{m4} & \ddots & W_{mn} \\ {W_{11}}^{0} & {W_{12}}^{0} & {W_{13}}^{0} & {W_{14}}^{0} & \vdots & {W_{1n}}^{0} \\ {W_{21}}^{1} & {W_{22}}^{1} & {W_{23}}^{1} & {W_{24}}^{1} & \vdots & {W_{2n}}^{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ {W_{m1}}^{n} & {W_{m2}}^{n} & {W_{m3}}^{n} & {W_{m4}}^{n} & & {W_{mn}}^{n} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} X_{11} \\ X_{21} \\ \vdots \\ \vdots \\ X_{mn} \\ {X_{11}}^{0} \\ {X_{22}}^{1} \\ \vdots \\ \vdots \\ {X_{mn}}^{n} \end{pmatrix} \quad (5)$$

where W_{is} and ${W_{is}}^{*}$ are the coefficients of a_{is} given as

$\mathrm{W}_{11}, \mathrm{W}_{12}, \mathrm{W}_{13}, \cdots \mathrm{W}_{1n} = \eta(t_1) + \mu_1(t_1)\tau(t_1) + \mu_2(t_1)\varsigma(t_1) + \mu_3(t_1)\xi(t_1) + \mu_4\,\gamma(t_1) + \mu_5(t_1)\chi(t_1) + \mu_6(t_1)\tau(t_1) + \omega(t_1)\omega(t_1)$

$\mathrm{W}_{21}, \mathrm{W}_{22}, \mathrm{W}_{23}, \cdots \mathrm{W}_{2n} = \eta(t_2) + \mu_1(t_2)\tau(t_2) + \mu_2(t_2)\varsigma(t_2) + \mu_3\xi(t_2) + \mu_4\,\gamma(t_2) + \mu_5(t_2)\chi(t_2) + \mu_6(t_2)(t_2)\tau(t_2) + \omega(t_2)\omega(t_2)$

$\mathrm{W}_{31}, \mathrm{W}_{32}, \mathrm{W}_{33}, \cdots \mathrm{W}_{3n} = \eta(t_3) + \mu_1(t_3)\tau(t_3) + \mu_2(t_3)\varsigma(t_3) + \mu_3\xi(t_3) + \mu_4(t_3)\,\gamma(t_3) + \mu_5(t_3)\chi t_3 + \mu_6(t_3)\tau(t_3) + \omega(t_3)\omega(t_3)$

${\mathrm{W}_{11}}^{0}, {\mathrm{W}_{12}}^{0}, {\mathrm{W}_{13}}^{0}, \cdots {\mathrm{W}_{1n}}^{0}$ are values of $v^{i}(a)$ and $v^{i}(b)$, and X_{is} are values of $f(t_i)$.
Let equation (5) be:

$$G(t_i)A = B(t_i) \quad (6)$$

Where $G(t_i) = \begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} & & W_{1n} \\ W_{21} & W_{22} & W_{23} & W_{24} & \cdots & W_{2n} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ W_{m1} & W_{m2} & W_{m3} & W_{m4} & \ddots & W_{mn} \\ {W_{11}}^{0} & {W_{12}}^{0} & {W_{13}}^{0} & {W_{14}}^{0} & \vdots & {W_{1n}}^{0} \\ {W_{21}}^{1} & {W_{22}}^{1} & {W_{23}}^{1} & {W_{24}}^{1} & \vdots & {W_{2n}}^{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ {W_{mn}}^{n} & {W_{m2}}^{n} & {W_{m3}}^{n} & {W_{m4}}^{n} & & {W_{mn}}^{n} \end{pmatrix}, A = \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ a_n \end{pmatrix}, B(t_i) = \begin{pmatrix} X_{11} \\ X_{21} \\ \vdots \\ \vdots \\ X_{mn} \\ {X_{11}}^{0} \\ {X_{22}}^{1} \\ \vdots \\ \vdots \\ {X_{mn}}^{n} \end{pmatrix}$

Multiply both sides of equation (7) by $G(t_i)^{-1}$ gives

$$A = G(t_i)^{-1}B(t_i) \quad (7)$$

The sought-after approximate solution is achieved by solving Equation (7) and then substituting the values of the unknown constants into the assumed approximation.

Numerical Examples
Example 4.1 [17]:
Consider the sixth Order Boundary Value Problem

$$v^6(t) = -e^{-t}v(t) - 720 + (t - t^2)^3 e^{-t} - (24 + 11t + t^3)e^t, \qquad 0 \le x \le 1,$$

Subject to the boundary conditions

$$v(0) = 0, v'(0) = 0, v''(0) = 0$$
$$v(1) = 0, v'(1) = 0, v''(1) = 0$$

With the exact solution $v(t) = t^3(1 - t)^3$

The method described above yielded the following unknown constants:

$a_0 = 0.00490799343032222$, $a_1 = 0.00363337400941077$, $a_2 = -0.00366616558853217$, $a_3 = -0{,}00145845725661431$, $a_4 = 0.00146489920132309$, $a_5 = 0.000243247309597905$, $a_6 = -0.000244053037691126$, $a_7 = 2.09204234463831 \times 10^{-8}$, $a_8 = 3.30229265055337 \times 10^{-9}$, $a_9 = 2.79885487548571 \times 10^{-10}$, $a_{10} = 6.01190492266665 \times 10^{-12}$

Thus, the approximate solution is given as;

$$\begin{aligned} v(t) = {} & 0.00002575126406t + 0.00004449205069 + 2.998813827x^5 - 1.000001130t^6 \\ & + 0.00001448668102t^7 - 0.00003512331024t^8 + 0.00004342658996t^9 \\ & + 0.000006303939217t^{10} - 2.997471404t^4 + 0.9985116227t^3 \\ & - 0.000003337748543t^2 \end{aligned}$$

Table 1. Shows numerical outcomes for example 4.1 at n=10

t	Exact	Approximate Solution n=10	[17] Absolute Error n=10	Absolute Error of proposed method n=10
0.0	0.000000	0.00004449205069	-	4.449E-05
0.1	0.000729	0.00077485313520	2.25E-04	4.585E-05
0.2	0.004096	0.0041375348600	7.36E-04	4.153E-05
0.3	0.009261	0.00929093143700	1.28E-03	2.993E-05
0.4	0.013824	0.01383666358000	1.68E-03	1.266E-05
0.5	0.015625	0.01561817254000	1.83E-03	6.827E-06
0.6	0.013824	0.01379936125000	1.68E-03	2.464E-05
0.7	0.009261	0.00922335963700	1.28E-03	3.764E-05
0.8	0.004096	0.00922335963700	7.36E-04	4.444E-05
0.9	0.000729	0.00068316123630	2.25E-04	4.584E-05
1	0.000000	-0.00004441255146	-	4.441E-05

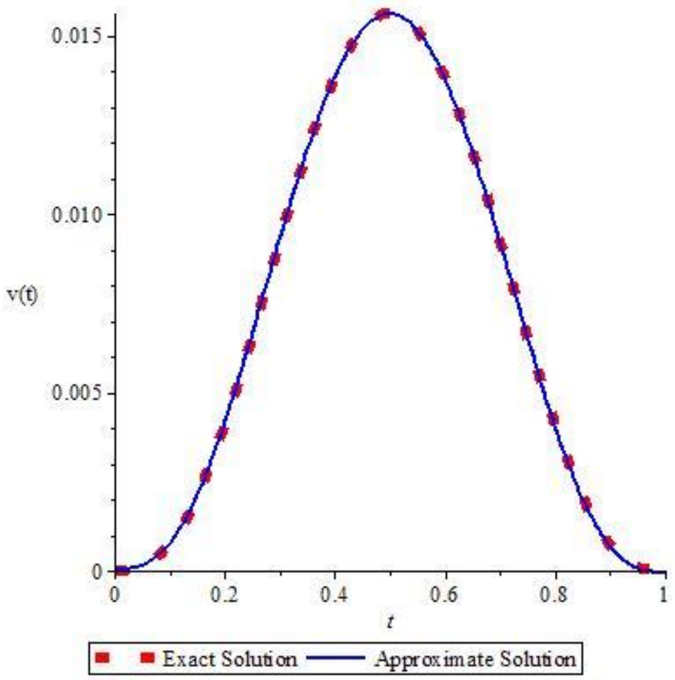

Figure 1. Demonstrates the graphical results for Example 4.1's precise solution and approximation solution

Example 4.2 [17]:
Consider the sixth Order Boundary Value Problem

$$v^6(t) = -tv(t) - (24 + 11t + t^3)e^t, \qquad 0 \leq x \leq 1,$$

Subject to the boundary conditions
$v(0) = 0, v'(0) = 1, v''(0) = 1$

$v(1) = 0, v'(1) = e, v''(1) = -4e$

with the exact solution $V(t) = (1 - t)e^t$

The unknown constants are determined through the method described above:

$a_0 = 0.186363835481144, a_1 = -0.129257730476006, a_2 = -0.0770104498035282,$
$a_3 = -0.0227622168161328,\ a_4 = -0.00298134018729407, a_5 = -0.000255683190010569,$
$a_6 = -0.0000160672348446424, a_7 = -8.09529136914819 \times 10^{-7},$
$a_8 = -3.38989522492939 \times 10^{-8}, a_9 = -1.23436508172706 \times 10^{-9},$
$a_{10} = -3.96164106489073 \times 10^{-11}$

Therefore, the approximate solution is expressed as:

$v(t) = 1.000039381t + 0.0001169246131 - 0.1267651183t^5 - 0.03333581402t^6 -$
$0.006917808190t^7 - 0.001243614852t^8 - 0.0001262625174t^9 - 0.00004154081741t^{10} -$
$0.3289484951t^4 - 0.5028946860t^3 - 1.014260607\times 10^{-7}t^2$

Table 2. Shows numerical outcomes for example 4.2 at n=10

t	Exact	Approximate Solution n=10	Absolute Error n=10 [17]	Absolute Error of the proposed method n=10
0.0	0.00000000000	0.0001169246131	-	1.169E-04
0.1	0.09946538262	0.0995837725200	3.81E-05	1.184E-04
0.2	0.19542444130	0.1955325397000	1.59E-04	1.081E-04
0.3	0.28347034970	0.2835521705000	3.41E-04	8.182E-05
0.4	0.35803792750	0.3580795469000	5.33E-04	4.162E-05
0.5	0.41218031780	0.4121740514000	6.74E-04	6.266E-06
0.6	0.43730851200	0.4372549752000	7.08E-04	5.354E-05
0.7	0.42288806850	0.4227961197000	6.08E-04	9.195E-05
0.8	0.35608654850	0.3559711134000	3.91E-04	1.154E-04
0.9	0.22136428000	0.2212420603000	1.35E-04	1.222E-04
1	0.00000000000	-0.0001169323739	-	1.169E-04

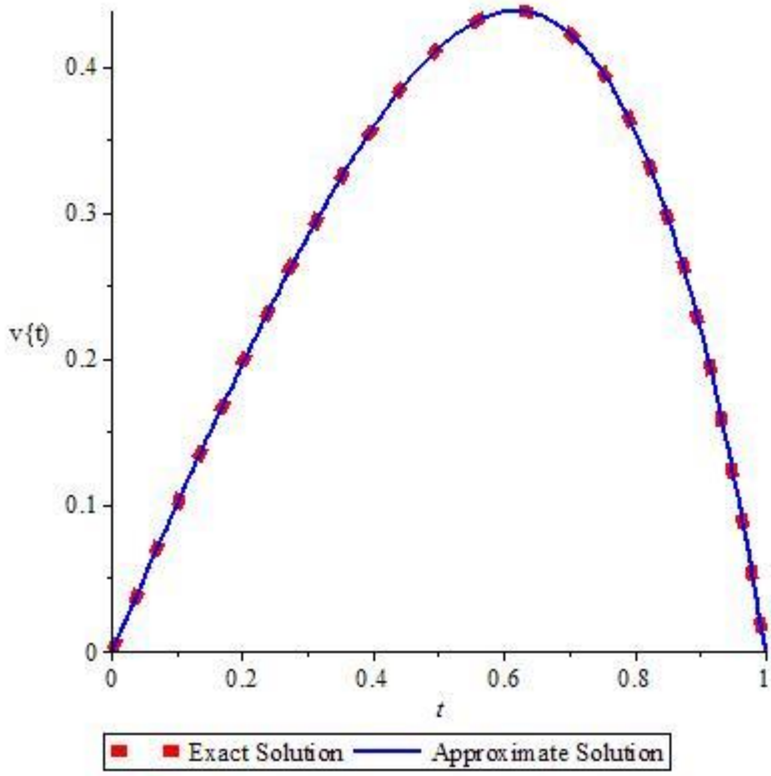

Figure 2. Demonstrates the graphical results for Example 4.2's precise solution and approximation solution

Conclusion

This work successfully employs the suggested approach to solve numerically six-order boundary value problems with fourth-kind shifted Chebyshev polynomials. The correctness and effectiveness of the method are demonstrated numerically using tables and figures. The proposed technique outperformed the method of [17] at all points, as can be seen from Example 1. It can also be argued that the proposed method marginally outperformed the method of [17] at points 0.3, 0.4, 0.5, 0.6, and 0.7 in example 2. Excellent agreement between the approximation solutions' graphs and the exact solutions can be seen in Figures 1–2. The outcomes of this study recommend the proposed strategy for resolving additional boundary value issues after taking the aforementioned factors into account.

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