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P-Convexity Property in Musielak-Orlicz Function Space of Bohner Type

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Abstracts

In this paper, we described about Musielak-Orlicz function spaces of Bochner type. It has been obtained that Musielak-Orlicz function space $L_\phi(\mu, X)$ of Bochner type becomes a Banach space. It is described also about P-convexity of Musielak-Orlicz function space $L_\phi(\mu, X)$ of Bochner type. It is proved that the Musielak-Orlicz function space $L_\phi(\mu, X)$ of Bochner type is P-convex if and only if both spaces L_ϕ and X are P-convex..

Key words: Musielak-Orlicz Function; Musielak-Orlicz Functio; Bochner type; P-Convex.

1. Introduction

The Orlicz space was first introduced in 1931 by W. Orlicz [1]. Orlicz space theory has a very important role and has been widely applied to various branches of mathematics, one of them on the issue of Optimal Control. The development and refinement of the Orlicz space itself is also progressing very rapidly, one them were Musielak and Orlicz [2] which develops a functional space generated by a modular that having convex properties. In this case, $\widetilde{I}_\phi(f) = \int_T \phi(t, \|f(t)\|_X) d\mu$ the modular convex generates the Musielak-Orlicz function spaces of Bochner-type, which is also the Banach space [3,4].

The convexity and reflexive properties of the Banach space also has been widely developed by many mathematicians. Yining, et.al [5,6] in their paper entitled "P-convexity and reflexivity of Orlicz spaces" proved that for Orlicz spaces reflexivity is equivalent to P-convexity. The same result for the Musielak-Orlicz sequence and function spaces were obtained by Kolwicz and Pluciennik [7-9].

2. Auxiliary Lemmas

Definition 1

*Given $f: T \to \mathbf{R}^*$ is μ-measurable function. Function f said to be **μ-integrable**, if there exist sequence function $\{f_n\}$ such that $f_n(x) \to f(x)$ a.e. $x \in T$ and for every $\varepsilon > 0$ there exist natural number n such that $\int_T |f_i(x) - f_j(x)| \mu(dx) < \varepsilon$, for every $i, j \geq n$. Hence, the finite value of $\lim_{n\to\infty} \int_T f_n(x)\mu(dx)$, is called **Lebesgue integral** of function f and denoted by $\int_T f(x)\mu(dx)$ or $\int_T f\, d\mu$.*

Given measurable set X. The set of all μ-measurable functions from X to $\mathbf{R}^*$ denoted by $M(X)$. It can be proved that $M(X)$ is a linear space. For $1 \leq p < \infty$, defined

$$L^p(X) = \left\{ f \in M(X) \middle| \int_X |f|^p d\mu < \infty \right\}.$$

In another word, $L^p(X)$ is a set of all measurable functions $f \in M(X)$, such that $|f|^p$ μ-integrable in *X*.

Given f extended real valued function on measurable set X. Supremum essensial f on E defined by

$$ess\sup\{|f(x)| : x \in X\} = M \Leftrightarrow \exists A \subset X, \mu(A) = 0 \ni \sup\{|f(x)| : x \in X - A\} = M.$$

Then, we defined $L^\infty(X)$ the set of all measurable functions by the formula

$$L^\infty(X) = \{ f : ess\sup\{|f(x) : x \in X|\} < \infty \}.$$

Definition 2

Given linear space T

Non-negatif function $\rho : T \to [0, \infty)$ is called **modular** *on T if for every $x, y \in T$ this conditions below apply*

(M1) $\rho(x) = 0 \Leftrightarrow x = \theta$,

(M2) $\rho(-x) = \rho(x)$,

(M3) $\rho(\alpha x + \beta y) \leq \rho(x) + \rho(y)$ *if* $\alpha, \beta \in [0,1]$ *with* $\alpha + \beta = 1$.

Linear space T that completed with modular is called **modular** space and denoted by (T, ρ).

A set $B \subseteq Y$, with Y linear space, is called **convex set** if for each $x, y \in B$ and $\alpha, \beta \in [0,1]$ with $\alpha + \beta = 1$, is applicable $\alpha x + \beta y \in B$. Function $f : B \to \mathbf{R}$ is called **convexs**, if B convexs and for every $x, y \in B$ and $\alpha, \beta \in [0,1]$ with $\alpha + \beta = 1$ be valid $f(\alpha x + \beta y) \leq \alpha f(x) + \beta f(y)$. Furthermore, the modular ρ is called comply character convexs if ρ is fungtion of convexs. On the next discussion, the meaning modular is the modular that comply of character convexs, Unless otherwise stated.

Theorem 3

(i) If $\alpha_1, \alpha_2 \in \mathbf{R}$ with $0 < \alpha_1 \leq \alpha_2$, then $\rho(\alpha_1 x) \leq \rho(\alpha_2 x)$ for every $x \in X$.

If $\rho(x) < \varepsilon$ for every $\varepsilon > 0$, then $x = \theta$.

Definition 4

Given *linear space T.*

*Function $\phi : T \times \mathbf{R} \to [0, \infty)$ is called **Musielak – Orlicz function** if:*

(1) $\phi(t,u) = 0 \Leftrightarrow u = 0$, *for every* $t \in T$,

(2) $\phi(t,-u) = \phi(t,u)$

(3) $\phi(t,.)$ *continu*

(4) $\phi(t,.)$ *increase on* $(0,\infty)$

(5) $\phi(\cdot,u)$ *measurable, for each* $u \in \mathbf{R}$

(6) $\phi(t,\cdot)$ *convex,*

(7) $\dfrac{\phi(t,u)}{u} \to 0$ *if* $u \to 0$ *on T.*

For the function of Musielak-Orlicz ϕ, is defained of function $I_\phi : L^0 \to [0, \infty)$ with

$$I_\phi(f) = \int_T \phi(t, f(t)) d\mu,$$

for every $f \in L^0$. So, can be shown that function I_ϕ is modular convexs. Then, defined space of function Musielak–Orlicz L_ϕ, with $L_\phi = \left\{ f \in L^0 : I_\phi(cf) < \infty \text{ for some } c > 0 \right\}$.

Forthermore, for every of the function Musielak–Orlicz ϕ, is defained of function $\phi^* : T \times \mathbf{R} \to [0, \infty)$, with
$$\phi^*(t,v) = \sup_{u>0}\{u|v| - \phi(t,u)\},$$
for every $v \in \mathbf{R}$ and $t \in T$. Can be shown that function ϕ^* is function of Musielak-Orlicz.

Theorem 5 *For every function of Musielak-Orlicz* ϕ*, be valid* $uv \le \phi(t,u) + \phi^*(t,v)$*,* $u, v \ge 0$*, for every* $t \in T$.

Definition 6 *Function of Musielak – Orlicz* ϕ *is called comply* **condition -** Δ_2*, writes* $\phi \in \Delta_2$*, if that constanta* $k > 0$ *and* $u_0 \ge 0$ *that* $\phi(t,2u) \le k\phi(t,u)$ *, for every* $t \in T$ *and* $u \ge u_0$.

Fathermore, defined function $\widetilde{I}_\phi : L^0(T,X) \to (0,\infty)$, with $\widetilde{I}_\phi(f) = \int_T \phi(t, \|f(t)\|_X)\, d\mu$, for every $f \in L^0(T,X)$. That, can be shown that function $\widetilde{I}_\phi$ is modular. For the function of Musielak-Orlicz ϕ, defained $L_\phi(\mu, X) = \left\{ f \in L^0(T,X) : \|f(.)\|_X \in L_\phi \right\}$.

The defained $\|.\| : L_\phi(\mu, X) \to \mathbf{R}$, with $\|f\| = \left\| \|f(.)\|_X \right\|_\phi$, for every $f \in L_\phi(\mu, X)$, can be shown that $(L_\phi(\mu, X), \|.\|)$ is normed space.

Theorem 7 *Space norm* $(L_\phi(\mu, X), \|.\|)$ *is space Banach*.

Furthermore, the space function $L_\phi(\mu, X)$ is called **space of function Musielak – Orlicz type Bochner**.

3. Main Result

Definition 8 Given the space norm $(X, \|.\|_X)$. *The set* $S(X) = \{x \in X : \|x\|_X = 1\}$ *is called area with center 0 and set* $U(X) = \{x \in X : \|x\|_X \le 1\}$ *is called disebut* **closed unit ball**.

Definition 9 *The space Banach X is called reflexive if for a* $\varepsilon < 0$ *there* δ *so as* $\|x - y\| < \varepsilon$ *, for* $x, y \in U(X)$ *with* $\left\| \frac{1}{2}(x+y) \right\| > 1 - \delta$.

Definition 10 *The space norm linear X is called* ***P–convexs****, if that* $\varepsilon > 0$ *and* $n \in N$ *so for every* $x_1, x_2, ..., x_n \in S(X)$*, be valid* $\min_{i \ne j; i, j \le n} \|x_i - x_j\|_X \le 2(1 - \varepsilon)$.

Lemma 11 *The space Banach* X *P–convexs if and only if that* $n_0 \in N$ *and* $\delta_0 > 0$ *so for every* $x_1, x_2, ..., x_{n_0} \in X \setminus \{0\}$ *that integers* i_0, j_0 *so be valid*
$$\left\| \frac{x_{i_0} - x_{j_0}}{2} \right\|_X \le \frac{\|x_{i_0}\|_X + \|x_{j_0}\|_X}{2} \left(1 - \frac{2\delta_0 \min\{\|x_{i_0}\|_X, \|x_{j_0}\|_X\}}{\|x_{i_0}\|_X + \|x_{j_0}\|_X} \right)$$

Theorem 12 *To every of function Musielak-Orlicz* ϕ *be valid, if* $\phi^* \in \Delta_2$ *then for any* $a > 1$ *that* $\xi > 1$ *so* $\phi\left(t, \frac{\xi}{a} u\right) \le \frac{\xi^{-1}}{a} \phi(t,u)$ *for every* $u \in \mathbf{R}$, $t \in T$.

Lemma 13 *Be discovered* ϕ *and* ϕ^* *meet the conditions* Δ_2.

For every $\varepsilon \in (0,1)$ *that of function is measurable* $h_\varepsilon : T \to \mathbf{R}^+$ *with* $I_\phi(h_\varepsilon) < \varepsilon$*, number* $a(\varepsilon) \in (0,1)$ *and* $\gamma = \gamma(a(\varepsilon)) \in (0,1)$ *such that for* μ *h.d.* $t \in T$ *be valid*

$\phi\left(t,\frac{u+v}{2}\right)\leq\frac{1-\gamma}{2}\left|\phi(t,u)+\phi(t,v)\right|$ *for every* $u\geq h_\varepsilon(t)$ *and* $\left|\frac{v}{u}\right|<a$.

Lemma 14 *If* $\phi\in\Delta_2$*, that for every* $\alpha\in(0,1)$ *there is an undeveloped sequence of measurable sets of up to* $\{B_m^\alpha\}$*, so* $\mu\left(T\setminus\bigcup_{m=1}^{\infty}B_m^\alpha\right)=0$ *and for every* $m\in N$*, there* $k_m^\alpha>2$ *so* $\phi(t,2u)\leq k_m^\alpha\phi(t,u)$ *for* μ a.e.*,* $t\in B_m^\alpha$ *and for every* $u\geq\alpha f(t)$*,there* f *of condition* Δ_2.

.**Lemma 15** *If* $\phi\in\Delta_2$*,then for every* $\varepsilon\in(0,1)$ *there is a measurable function* $g_\varepsilon:T\to\mathbf{R}^+$ *and* $k_\varepsilon>2$ *so* $I_\phi(g_\varepsilon)<\varepsilon$ *and* $\phi(t,2u)\leq k_\varepsilon\phi(t,u)$*, for* μ a.e.*,* $t\in T$ *,and* $u\geq g_\varepsilon(t)$.

Theorem 16 *If space Banach X P-convexs, then X reflextif.*

Theorem 17 *Be discovered* ϕ *function Musielak – Orlicz and X space Banach.*
Then the following statements are equivalent:
(a) $L_\phi(\mu,X)$ *P-convex,*
(b) L_ϕ *and X P-convex,*
(c) L_ϕ *reflectif dan X P-convex,*
(d) X P-convex, $\phi\in\Delta_2$ *and* $\phi^*\in\Delta_2$.

Evidence:

$(a)\Rightarrow(b)$

Be discovered $L_\phi(\mu,X)$ P-convex. Because of the space L_ϕ dan X embedded isometrically to $L_\phi(\mu,X)$ and characteristics P-convex as well apply on subspace, then obtained L_ϕ and X P-convex.

$(b)\Rightarrow(c)$

Be discovered L_ϕ and X P-convex. According to theorem 9, obtained L_ϕ reflexive.

$(c)\Rightarrow(d)$

The characteristics of reflexsive on the space funcion of Musielak-Orlicz L_ϕ ekuivalen with $\phi\in\Delta_2$ and $\phi^*\in\Delta_2$.

$(d)\Rightarrow(a)$

Be discovered X P-convex, $\phi\in\Delta_2$ and $\phi^*\in\Delta_2$. Can be chosen $n_0\in N$.

Then, for every $t\in T$ defined

$$f(t)=\max\left\{h_{\frac{1}{4n_0}}(t),g_{\frac{1}{4n_0}}(t)\right\},$$

there of function $h_{\frac{1}{4n_0}}$ and $g_{\frac{1}{4n_0}}$ is function measureble with $\varepsilon=\frac{1}{4n_0}$.

as a result, $I_\phi(f)<\frac{1}{2n_0}$. Can be chosen $\alpha=a$. Because $\phi\in\Delta_2$, then obtained $I_\phi\left(\frac{1}{a}f\right)<\infty$.

Selected set $B_{m_0}^a$ so fulfilling

$$\int_{T\setminus B_{m_0}^a}\phi\left(t,\frac{f(t)}{a}\right)d\mu<\frac{1}{2n_0}.$$

Selected $l\in\mathbf{R}$ so $\frac{1}{a}\leq 2^l$. There are numbers $k_{m_0}^a>2$ so

$$\phi\left(t,\frac{1}{a}v\right)\leq\left(k_{m_0}^a\right)^l\phi(t,v)\text{ for }\quad\mu-a.e.\ t\in B_{m_0}^a,$$

when $v\geq a\,f(t)$.

Then, chosen $\frac{v}{a}=u$ and $\frac{1}{\left(k_{m_0}^a\right)^l}=\beta(a,m_0)=\beta_{m_0}$. Obtained

$$\phi(t,au)\geq\beta_{m_0}\phi(t,u),$$

For $\mu-h.d.\ t\in B_{m_0}^a$, and for every $u\geq f(t)$.

So, obtained $\phi\left(t,\frac{1}{a}v\right)\le (k_\varepsilon)^l\phi(t,v)$ for $\mu-a.e.\ t\in T$, and $v\ge f(t)$, then $k_\varepsilon = k_{\frac{1}{4n_0}}$.

In the same way, obtained $\phi(t,au)\ge \beta\phi(t,u)$,

For $\mu-h.d.\ t\in T$, and for every $u\ge \frac{f(t)}{a}$.

Furthermore, it will be shown that there are numbers $r_1\in(0,1)$ so for every $x_1,x_2,\dots,x_{n_0}$ space group Banach X and to $\mu-a.e.t\in T_M$ obtained

$$\sum_{i=1}^{n_0}\sum_{j=i}^{n_0}\phi\left(t,\left\|\frac{x_i-x_j}{2}\right\|_X\right)\le \frac{n_0-1}{2}r_1\sum_{i=1}^{n_0}\phi\left(t,\|x_i\|_X\right)$$

,

With $T_M=\left\{t\in T:\max_{1\le i\le n_0}\{\|x_i\|_X\}\ge \frac{f(t)}{a}\right\}$.

Taken $x_1,x_2,\dots,x_{n_0}\in X$. For example k is index so $\|x_k\|_X=\max_{1\le i\le n_0}\{\|x_i\|_X\}$.

I. Suppose there $i_1\in\{1,2,\dots,n_0\}\setminus\{k\}$ so $\frac{\|x_{i_1}\|_X}{\|x_k\|_X}<a$.

Because $\|x_k\|\ge\frac{f(t)}{a}>f(t)$ for $\mu-a.e.t\in T_M$, so obtained

$$\phi\left(t,\left\|\frac{x_{i_1}-x_k}{2}\right\|_X\right)\le\phi\left(t,\frac{\|x_{i_1}\|_X+\|x_k\|_X}{2}\right)$$
$$\le\frac{1}{2}(1-\gamma)\left(\phi\left(t,\|x_{i_1}\|_X\right)+\phi\left(t,\|x_k\|_X\right)\right).$$

Based on characteristics convex of $\phi(t,.)$ for $\mu-a.e.\ t\in T$, obtained

$$\sum_{i=1}^{n_0}\sum_{j=i}^{n_0}\phi\left(t,\left\|\frac{x_i-x_j}{2}\right\|_X\right)\le\frac{n_0-1}{2}\sum_{i=1}^{n_0}\phi\left(t,\|x_i\|_X\right)-\frac{\gamma}{2}\left(\phi\left(t,\|x_{i_1}\|_X\right)+\phi\left(t,\|x_k\|_X\right)\right)$$

$$\le\frac{n_0-1}{2}\sum_{i=1}^{n_0}\phi\left(t,\|x_i\|_X\right)-\frac{\gamma}{2n_0}\left(n_0\phi\left(t,\|x_k\|_X\right)\right)$$

$$\le\frac{n_0-1}{2}\sum_{i=1}^{n_0}\phi\left(t,\|x_i\|_X\right)-\frac{\gamma}{2n_0}\sum_{i=1}^{n_0}\phi\left(t,\|x_i\|_X\right)$$

$$=\frac{n_0-1}{2}\left(1-\frac{\gamma}{n_0(n_0-1)}\right)\sum_{i=1}^{n_0}\phi\left(t,\|x_i\|_X\right),$$

for $\mu-a.e.\ t\in T_M$.

II. It is assumed that for all $i\ne k$ apply $\frac{\|x_{i_1}\|_X}{\|x_k\|_X}\ge a$.

Then $\|x_i\|>0$, for every $i\ne k$. Can be taken i_0, j_0. and assumed that

$$a<\frac{\|x_{i_1}\|_X}{\|x_{j_0}\|_X}\le\frac{1}{a}.$$

On the other hand, haved

$$a>\frac{\|x_{i_1}\|_X}{\|x_{j_0}\|_X}\ge\frac{\min\{\|x_{i_0}\|_X,\|x_{j_0}\|_X\}}{\max\{\|x_{i_0}\|_X,\|x_{j_0}\|_X\}}\ge\frac{\min\{\|x_{i_0}\|_X,\|x_{j_0}\|_X\}}{\|x_k\|_X}$$

.

So it's a contradiction. So obtained

$$\left\|\frac{x_{i_0}-x_{j_0}}{2}\right\|_X\le\left(1-\frac{2\delta a}{1+a}\right)\frac{\|x_{i_0}\|_X+\|x_{j_0}\|_X}{2}.$$

Based on characteristics convex of $\phi(t,.)$ for $\mu-a.e.\ t\in T$, obtained

$$\phi\left(t,\left\|\frac{x_{i_0}-x_{j_0}}{2}\right\|_X\right)\le\frac{1}{2}(1-\alpha)\left(\phi\left(t,\|x_{i_0}\|_X\right)+\phi\left(t,\|x_{j_0}\|_X\right)\right)$$

,

With $\alpha=\frac{2\delta a}{1+a}\in(0,1)$. so, obtained

$$\sum_{i=1}^{n_0}\sum_{j=i}^{n_0}\phi\left(t,\left\|\frac{x_i-x_j}{2}\right\|_X\right)\le\frac{n_0-1}{2}\sum_{i=1}^{n_0}\phi\left(t,\|x_i\|_X\right)-\frac{\alpha}{2}\left(\phi\left(t,\|x_{i_0}\|_X\right)+\phi\left(t,\|x_{j_0}\|_X\right)\right)$$

$$\le\frac{n_0-1}{2}\sum_{i=1}^{n_0}\phi\left(t,\|x_i\|_X\right)-\alpha\phi\left(t,a\|x_k\|_X\right)$$

$$\le\frac{n_0-1}{2}\sum_{i=1}^{n_0}\phi\left(t,\|x_i\|_X\right)-\frac{\alpha\beta}{n_0}\left(n_0\phi\left(\|x_k\|_X\right)\right)$$

$$\leq \frac{n_0-1}{2}\sum_{i=1}^{n_0}\phi\left(t,\|x_i\|_X\right)-\frac{\alpha\beta}{n_0}\sum_{i=1}^{n_0}\phi\left(t,\|x_i\|_X\right)$$

$$=\frac{n_0-1}{2}\left(1-\frac{2\alpha\beta}{n_0(n_0-1)}\right)\sum_{i=1}^{n_0}\phi\left(t,\|x_i\|_X\right),$$

for $\mu-a.e.\ t\in T_M$.

Defined

$$r_1=\max\left\{1-\frac{\gamma}{n_0(n_0-1)},1-\frac{2\alpha\beta}{n_0(n_0-1)}\right\}.$$

Then the number can be determined $r_2\in(0,1)$ so be valid

$$\sum_{i=1}^{n_0}\sum_{j=i}^{n_0}\phi\left(t,\left\|\frac{x_i-x_j}{2}\right\|_X\right)\leq\frac{n_0-1}{2}r_2\sum_{i=1}^{n_0}\phi\left(t,\|x_i\|_X\right)$$

, for every $x_1,x_2,\ldots,x_{n_0}$ component space Banach X and for $\mu-h.d.\ t\in B^a_{m_0}$ comply $maks_{1\leq i\leq n_0}\{\|x_i\|_X\}\geq f(t)$. Then proved true with

$$r_2=\max\left\{1-\frac{\gamma}{n_0(n_0-1)},1-\frac{2\alpha\beta_{m_0}}{n_0(n_0-1)}\right\}.$$

Taken $f_1,f_2,\ldots,f_{n_0}\in S(L_\phi(\mu,X))$ and defained

$$E=\left\{t\in T:\sum_{i=1}^{n_0}\phi\left(t,\|f_i(t)\|_X\right)\geq n_0\phi(t,f(t))\right\}.$$

Obviously that, $\max_{1\leq i\leq n_0}\left\{\|f_i(t)\|_X\right\}\geq f(t)$ for every $t\in E$. Furthermore E divided into the following two subsets:

$$E_1=\left\{t\in T:\max_{1\leq i\leq n_0}\left\{\|f_i(t)\|_X\right\}\geq\frac{f(t)}{a}\right\},\text{ and}$$

$$E_2=\left\{t\in T: f(t)\leq\max_{1\leq i\leq n_0}\left\{\|f_i(t)\|_X\right\}<\frac{f(t)}{a}\right\}.$$

Defined set E_{21} dan E_{22} as follows $E_{21}=E_2\cap B^a_{m_0}$, and $E_{22}=E_2\setminus B^a_{m_0}$.

Obtained,

$$\sum_{i=1}^{n_0}\sum_{j=i}^{n_0}\phi\left(t,\left\|\frac{f_i(t)-f_j(t)}{2}\right\|_X\right)\leq\frac{1}{n_0}\binom{n_0}{2}\sum_{i=1}^{n_0}\phi\left(t,\|f_i(t)\|_X\right)$$

,

for $\mu-a.e.\ t\in E_1\cup E_2$, with $r=\max\{r_1,r_2\}$. obviously $r\in(0,1)$. Furthermore, from the define set E and function f, obtained

$$\sum_{i=1}^{n_0}\tilde{I}_\phi\left(f_i\chi_{T\setminus E}\right)<\frac{1}{2}.$$

taken $t\in E_{22}$, then obtained

$$\sum_{i=1}^{n_0}\tilde{I}_\phi\left(f_i\chi_{E_{22}}\right)=\sum_{i=1}^{n_0}\int_{E_2\setminus B^a_{m_0}}\phi\left(t,\|f_i(t)\|_X\right)d\mu$$

$$\leq\int_{E_2\setminus B^a_{m_0}}n_0\varphi\left(t,\max_{1\leq i\leq n_0}\left\{\|f_i(t)\|_X\right\}\right)d\mu$$

$$<\int_{E_2\setminus B^a_{m_0}}n_0\phi\left(t,\frac{f(t)}{a}\right)d\mu$$

$$\leq\int_{T\setminus B^a_{m_0}}n_0\phi\left(t,\frac{f(t)}{a}\right)d\mu<\frac{1}{2}.$$

Obtained

$$\sum_{i=1}^{n_0}\tilde{I}_\phi\left(f_i\chi_{T\setminus(E_1\cup E_{21})}\right)=\sum_{i=1}^{n_0}\tilde{I}_\phi\left(f_i\chi_{T\setminus E}\right)+\sum_{i=1}^{n_0}\tilde{I}_\phi\left(f_i\chi_{E_{22}}\right)<1$$

.

because $\|f_i\|=1$ for $i=1,2,\ldots,n_0$ and $\phi\in\Delta_2$, then $\tilde{I}_\phi(f_i)=1$ for $i=1,2,\ldots,n_0$. as a result,

$$\sum_{i=1}^{n_0}\tilde{I}_\phi\left(f_i\chi_{E_1\cup E_{21}}\right)\geq n_0-1.$$

obtained

$$\sum_{i=1}^{n_0}\sum_{j=i}^{n_0}\tilde{I}_\phi\left(\frac{1}{2}(f_i-f_j)\right)=\sum_{i=1}^{n_0}\sum_{j=i}^{n_0}\tilde{I}_\phi\left(\frac{1}{2}(f_i-f_j)\chi_{T\setminus(E_1\cup E_{21})}\right)+\sum_{i=1}^{n_0}\sum_{j=i}^{n_0}\tilde{I}_\phi\left(\frac{1}{2}(f_i-f_j)\chi_{E_1\cup E_{21}}\right)$$

$$\leq\frac{1}{n_0}\binom{n_0}{2}\sum_{i=1}^{n_0}\tilde{I}_\phi\left(f_i\chi_{T\setminus(E_1\cup E_{21})}\right)+\frac{r}{n_0}\binom{n_0}{2}\sum_{i=1}^{n_0}\tilde{I}_\phi\left(f_i\chi_{E_1\cup E_{21}}\right)$$

$$= \frac{1}{n_0}\binom{n_0}{2}\left(\sum_{i=1}^{n_0}\tilde{I}_\phi(f_i) - \sum_{i=1}^{n_0}\tilde{I}_\phi\left(f_i\chi_{E_1\cup E_{21}}\right)\right) + \frac{r}{n_0}\binom{n_0}{2}\sum_{i=1}^{n_0}\tilde{I}_\phi\left(f_i\chi_{E_1\cup E_{21}}\right)$$

$$= \binom{n_0}{2}\left(1 - \frac{1-r}{n_0}\sum_{i=1}^{n_0}\tilde{I}_\phi\left(f_i\chi_{E_1\cup E_{21}}\right)\right)$$

$$\leq \binom{n_0}{2}\left(1 - \frac{(1-r)(n_0-1)}{n_0} \leq \binom{n_0}{2}(1-p)\right),$$ with $p = \frac{(1-r)}{2}$.

So, be found $i_1, j_1 \in \{1,2,\ldots,n_0\}$ so that $\tilde{I}_\phi\left(\frac{1}{2}\left(f_{i_1} - f_{j_1}\right)\right) \leq 1-p$.

as a result, because $\phi \in \Delta_2$, obtained $\left\|\frac{1}{2}\left(f_{i_1} - f_{j_1}\right)\right\| \leq 1 - q(p)$, with $0 < q(p) < 1$.

obtained that space $L_\phi(\mu, X)$ P-convexs.

4. Conclusion

Modular $I_\phi(f) = \int_T \phi(t, f(t))d\mu$ generates the Musielak-Orlicz function space $L_\phi = \{f \in L^0 : I_\phi(cf) < \infty \ \ for \ \ some \ c > 0\}$. Furthermore, for every Musielak - Orlicz function ϕ, we define function $\phi^*(t,v) = \sup_{u>0}\{u|v| - \phi(t,u)\}$, which is also a Musielak - Orlicz function and it is called the complementary function in the sense of Young. Modular $\tilde{I}_\phi(f) = \int_T \phi(t, \|f(t)\|_X)d\mu$ generates the Musielak-Orlicz function spaces of Bochner type $L_\phi(\mu, X) = \{f \in L^0(T,X) : \|f(.)\|_X \in L_\phi\}$. Furthermore, $L_\phi(\mu, X)$ is a Banach space.

The Musielak-Orlicz function spaces of Bochner type $L_\phi(\mu, X)$ is P-convex if and only if both L_ϕ and *X* are P-convex. Furthermore, it is proved that reflexivity is equivalent to P-convexity.

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