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Simulation Nongravitational Dynamic of Cometary Orbit

Arsini[1], Joko Saefan[2], Muhammad Farchani Rosyid[3]
[1]Department of Physics Education, FITK, Universitas Islam Negeri Walisongo, Central Java, Indonesia
[2]Department of Physics Education, FPMIPA Universitas PGRI Semarang, Indonesia
[3]Department of Physics, FMIPA UGM, Yogyakarta, Indonesia

Corresponding author: arsiaulia62@yahoo.com
Telp. (+62-24-7601295)
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Abstracts

The tail formation of a comet near the sun leads to the situation in which the comet continually losses a part of its masses so that the mass of the comet decreases monotonically. A comet may also accrete the material encountered along its orbit so that its mass increases. Therefore, the mass of a comet can be regarded as a function of time. In this work we study simulation the dynamics of the orbit of a comet due to the lost of its mass along the formation of its tail and the material accretion along its orbit. Here, we assume that the comet under consideration is of the form of a ball and rotates so rapidly that the whole of its surface catches the radiation of the sun equally.

Keywords*:* Simulation, Comet, Dynamics, Non-gravitational*.*

1. Introduction

The term comet refers to the expanding atmosphere consisting of dust and neutral as well as ionized gas which appears around a body of small size (called nucleus) in an eccentric orbit around the sun. The atmosphere is called coma and formed when the nucleus moves near the sun. Due to the solar wind and solar radiation pressure, two kinds of tails are formed from the coma, i.e., a dust tail and an ion tail which are stretched about 10^4 kilometers to 10^8 kilometers along in the radial direction away from the sun. The cometary activities are therefore related to the sun.

In almost all proposed models of comet, a cometary nucleus is composed of a mixture of ices and dust particles. The coma of a comet is formed by the sublimation of ices of the nucleus due to incident solar radiation. The sublimation also frees meteorite dust particles. The atmosphere of a comet is therefore composed of neutral as well as photoionized gases and dust particles. The solar radiation pressure then pushes the dust particles away so that a tail of the comet which is composed of dust particles appears. While the ions of the coma are guided by magnetic field line frozen in the solar wind flowing radial away from the sun to form a tail composed of ions. Therefore, the tail formation of a comet around the sun leads to the situation in which the comet continually losses a part of its masses so that the total mass of the comet decreases monotonically. On the other hand, a comet can also accrete the material encountered along its orbit particularly when the comet passes around its aphelion so that its mass increases. Hence, the mass of a comet can be regarded as a function of time.

That the orbits of comets evolve is already realized by scientists. The evolution of cometary orbit can be either gravitational or nongravitational in nature. The gravitational evolutions are primarily due to the gravitational perturbations of the Jovian planets. However, we are not in the position to discusse such phenomena.

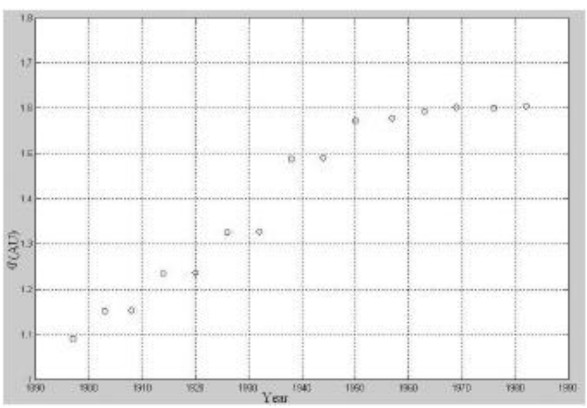

Figure 1. Perihelion distance vs. year of apparition of Comet P/Tempel-Swift [1]

Graph in Fig.1 is adapted from [1] and shows the orbital evolution of Comet P/Tempel-Swift which is obtained by observational data. Clearly, the perihelion distance q of Comet P/Tempel-Swift increases monotonically with its year of apparition.

Graph in Fig.2 adapted from [2] shows the orbital evolution of Comet 6P/d'Arrest. The perihelion distance q of the comet does not however evolve monotonically with its year of apparition.

The above mentioned evolution of cometary orbit is nongravitational in nature. In 1970, Marsden determined values of two constant nongravitational parameters A1 and A2 representing radial and transversal terms of nongravitational force in the equation of motion

$$\ddot{\mathbf{r}} + k^2 \frac{\mathbf{r}}{r^3} = \nabla R + A_1 g(r)\frac{\mathbf{r}}{r} + A_2 g(r)\frac{r\dot{\mathbf{r}} - \dot{r}\mathbf{r}}{h} + A_1 g(r)\frac{\mathbf{r}\times\dot{\mathbf{r}}}{h},$$

where R is the planetary disturbing function and k the Gaussian gravitational constant [3]. Marsden assumed that normal component of the forces to the orbital plane has a negligible influence on the orbital motion of short period comets.

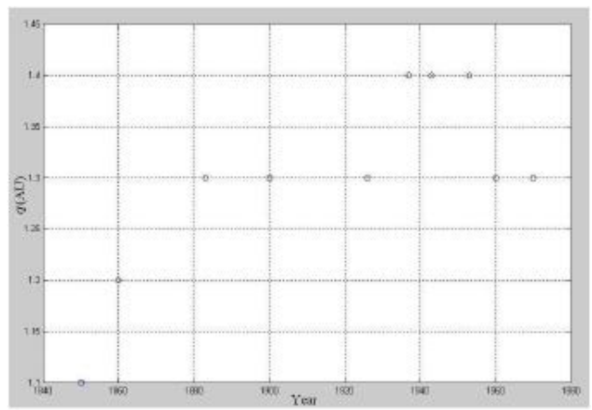

Figure 2. Perihelion distance vs. year of apparition of Comet 6P/de'Arrest [2]

In this work we also study the nongravitational dynamics of the cometary orbit due to mass loss along the formation of its tail. Here, we assume that the comet under consideration is of the form of a ball and rotates so rapidly that the whole of its surface catches the radiation of the sun equally. Following Marsden, we also assume that the components of all nongravitational forces normal to the orbital plane of the comet under consideration have a negligible influence on the orbital motion. Here, we derive an equation of motion based on the assumption that the Hamiltonian Least Action Principle is most fundamental and the mass of the comet under consideration is regarded as one of the general coordinates. By making use of the yielded equation we give an explanation of the orbital evolution of comets.

2. The Rate of Cometart Mass Loss

As mentioned above, the cometary mass loss is caused by the formation of the tail which is triggered by the sublimation of the ices of the nucleus. Thus, the rate of cometary mass loss
depends on the rate of the ices sublimation, the ionization of gas coma, the liberation of the dust particles, and the solar wind velocity.

2.1 *The Rate of the Ices Sublimation*

The source of energy for the cometary ices sublimation is solar radiation. The sublimation rate depends on intensity of the solar radiation falling on the nucleus surface and the Bond albedo (A) of the comet. The energy balance equation [4], between the input: solar radiation and the following losses: (i) thermal infrared radiation, (ii) sublimation of ices, and (iii) heat conduction into the nucleusinterior, namely

$$(1-A)\frac{F_* e^{-\tau}}{r^2}\pi R_N^2 = 2\pi R_N^2(1-A_{IR})\sigma T^4 + \frac{QL_S}{N_A} + 2\pi R_N^2 \kappa(T)\left|\frac{\partial T}{\partial z}\right|_{z=0}, \quad (1)$$

where (1 − A) is the fraction of incident radiation absorbed by the comet, AIR the Bond albedo in the infrared, $F^* = 3.16\times10^{-2}$calori·cm^{-2}·s^{-1} the solar constant, r heliocentric distance expressed in astronomical unit, σ the Stefan-Boltzmann constant, Q the total rate of sublimation, L_S the latent heat of sublimation expressed in calori·mol−1, $\kappa(T)$ thermal conductivity of cometary material, and τ the optical depth. The total rate of the sublimation Q is obtained as the solution of Equation (1).

2.2 *The Rate of Dust Particles Liberation*

The rate of the liberation of the dust particles depends on the scattering efficiency due to the radiation pressure (Q_{pr}). According to [5], the rate of dust particles liberation is given by

$$\dot{m}_d = \frac{\pi}{6} k C_{pr} Q_{pr} \int_0^{\infty} \frac{\Phi(t, 1-\mu)}{1-\mu} d(1-\mu), \quad (2)$$

where C_{pr} is a constant whose value is 1.19 × 10−3 kg.m^{-2}, k *a* dimensionless constant related to the flux of photon, Φ a distribution function, and 1 − μ a parameter defined by

$$1-\mu = \frac{C_{pr} Q_{pr}}{\rho_d d},$$

where ρ_d is the mass density of the dust and d the diameter of dust grain. The constant k appearing in Equation (2) means the mass loss rate of comet due to dust particles liberation inversely proportional to the quadrate of heliocentric distance.

2.3 The Rate of the Ionisation of Gas Coma

The cometary ionosphere is formed by photoionization of the neutral gases produced by the sublimation. In [6], the rate of the photoionization of neutral gases in coma is given by

$$\dot{n}_n = \frac{Q}{4\pi\lambda r_c^2}\exp\left(-\frac{r_c}{\lambda}\right),$$

where Q is the rate of ices sublimation, r_c cometocentric distance, and λ the ionisation length scale. From the rate of the photoionization, the mass loss rate of comet due to the formation of ion tail is given by

$$\dot{\rho}_{ion} = \frac{m_c Q}{4\pi\lambda r_c^2}\exp\left(-\frac{r_c}{\lambda}\right), \quad (3)$$

where m_c is the mean molecular mass.

2.4 The Solar Radiation Pressure

The solar radiation pressure is inversely proportional to the quadrate of heliocentric distance, i.e.

$$p_R(r) = \frac{F_*}{cr^2}, \quad (4)$$

where c is the speed of light in vacum.

3. Result and Discussion

Simulation Nongravitational Dynamics of Cometary Orbit

Several equations of motion in physics are derived from the variational principle. In classical mechanics, the principle manifests in the Hamiltonian Least Action (HLA) principle which describes the motion of monogenic mechanical system.In this study, we regard the principle as the most fundamental principle and derive the cometary equation of motion from the principle in which we take the influences of the cometary mass loss into account.

The fundamental facts of the celebrated variational principle are summarized in the following theorem [7]:

Theorem:

Let $F \in C^2([\ a,\ b\ ]\times R^d\times R^d,R)$ be a twice differentiable real-valued function defined on $[\ a,\ b\ ]\times R^d\times R^d$ and $u \in C^2([\ a,\ b\ ],\ R^d)$ a twice differentiable curve on Rd so that u(a) = u1 and u(b) = u2, where u1 and u2 is two fixed points in R^d. If the curve u minimalizes the integral

$$I(u)=\int_a^b F(t,u(t),\dot{u}(t))\,dt\,,$$

then the function *F* satisfies

$$\frac{d}{dt}\left(\frac{\partial F(t,u,\dot{u})}{\partial \dot{u}_\alpha}\right)-\frac{\partial F(t,u,\dot{u})}{\partial u_\alpha}=0\,, \qquad (5)$$

for α = 1, 2, ..., d.

In the HLA principle, the role of the function *F* is played by the Lagrangian function *L*. In general, when the continually change of cometary mass is abandoned, the appropriate Lagrangian function of the motion of a comet of mass m around the sun is given in spherical coordinate system by

$$L(t,r,\theta,\phi,\phi_R,\theta_R,\dot{r},\dot{\theta},\dot{\phi},\dot{\phi}_R,\dot{\theta}_R)= \frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2+r^2\sin^2\theta\dot{\phi}^2)+\frac{1}{2}\boldsymbol{\omega}\cdot\mathbf{I}\cdot\boldsymbol{\omega}+\frac{GM_*m}{r}+I_{JP}, \qquad (6)$$

where φ_R and θ_R are the angles related to the rotational degree of freedoms, $\boldsymbol{\omega}$ the angular velocity of its rotation, $\boldsymbol{I}$ its inertia tensor, and the last term represents the interaction of the comet with the Jovian planets. Since the size of the comet is too small compared with the size of its orbit, the second term of the Lagrangian can be neglected. Furthermore, we also assume that the orbit of the comet is far from the Jovian planets so that the last term can also be canceled out and no gravitational agent affects the orbit of the comet. From the symmetry argument, we know that the angular momentum of the comet conserves so that its orbit lies on a plane.

Now, let take the continually comet mass loss into account. If the cometary material liberation is of the same rate in all direction (since the rotation of the comet is fast enough and it is spheric, the assumtion is satisfied) then its orbit still lies on a plane. Let the orbital plane be the XY-plane of a cartesian coordinate system with the sun located in the origin and the angular momentum vector of the comet directs in the positive Z-axis. Therefore, the Lagrangian (6) can be written as

$$L(r,\phi,\dot{r},\dot{\phi})=\frac{1}{2}m(t)(\dot{r}^2+r^2\dot{\phi}^2)+\frac{GM_*m(t)}{r}.$$

If we regard the mass of the comet as a general coordinate, then the equation of motion of the comet can be derived from the above theorem where

$$F(t,u(t),\dot{u}(t))=L(m,r,\phi,\dot{m},\dot{r},\dot{\phi}). \qquad (7)$$

Thanks to Eq.(5), we have

$$m\ddot{r}+\dot{m}\dot{r}-mr\dot{\phi}^2+\frac{GM_*m}{r^2}=0\,, \qquad (8)$$

$$\frac{d}{dt}(mr^2\dot{\phi})=0\,, \qquad (9)$$

$$\dot{r}^2+r^2\dot{\phi}^2+\frac{2GM_*}{r}=0. \qquad (10)$$

Equation (9) expresses the conservation of the angular momentum of the comet. It means also the conservation of its inclination. The Kepler's second law however is not satisfied.

From the discussion in Section 2, it is fair to assume that the rate of mass loss of the comet is inversely proportional to the square of heliocentric distance and directly to its mass, i.e.

$$\dot{m} = -\frac{\alpha m}{r^2}, \qquad (11)$$

where α is positive constant. Hence, Equation (8) becomes

$$m\ddot{r} - \frac{\alpha m}{r^2}\dot{r} - mr\dot{\phi}^2 + \frac{GM_* m}{r^2} = 0. \qquad (8')$$

By making use of a small program (written in Delphi 7), we obtain Fig. 3 and 4 which describes the motion of a comet & obeying Equation (8'), (9), and (10) for m(0) =1, α = 1, r (0) & = 0.01, and φ = 0.

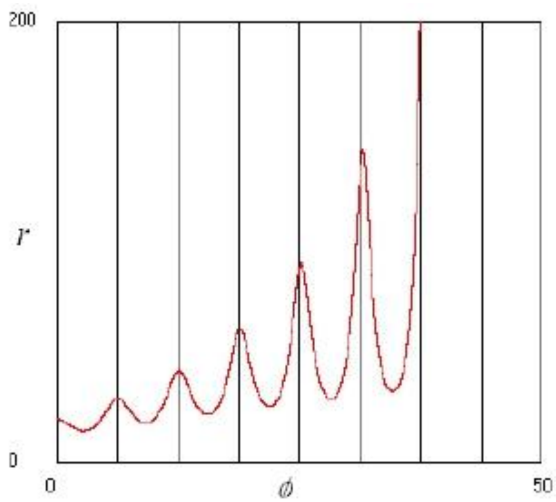

Figure 3. A visualization of the orbit of a comet obeying (8'), (9), and (10) on its orbital plane.

As depicted in Fig.3 and Fig.4, the perihelion and aphelion distance of comet increase monotonically with time (azimuthal angle). Fig.3 shows that the perihelion and aphelion point are not located in the X-axis anymore. They are also not located in the same radial line (with respect to the sun). This means,there is a continually angular shift of the aphelion and perihelion point. The vertical lines appearing in Fig. 4 are separated by an equal angular distance, i.e. the angular distance of the second aphelion from the first one. The figure shows therefore that the shift of aphelion and perihelion point is not linear.

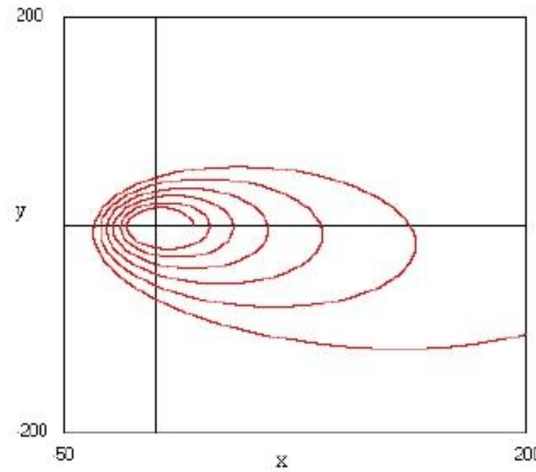

Figure 4. The heliocentric distance of a comet obeying (8'), (9), and (10) as a function of time. The vertical lines are separated by an equal angulardistance, i.e. the angular distance of the second aphelion from the first one.

4. Conclusion

The dynamics of cometary orbit can be either gravitational or nongravitational in nature. The total rate of cometary mass loss depends on the rate of the ices sublimation, the ionization of gas coma, the liberation of the dust particles, and the solar wind velocity. Therefore, the rate of mass loss of the comet is inversely proportional to the square of heliocentric distance and directly to its mass. The equation of motions derived from the Hamiltonian Least Action Principle in which the mass of the comet under consideration is regarded as one of the general coordinates have solutions with the perihelion and aphelion distance of comet increasing monotonically with the time. There is a continually angular shift of the aphelion and perihelion point. The shift is not linear. Simulation dynamics of cometary orbit programming by *Borland Delphi software*.

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