Fermion mass formulation in the Modified Left-Right Symmetry Model

Nurul Embun Isnawati, Istikomah, Muhammad Ardhi Khalif

1Department of Physics, Faculty Science and Technology, Universitas Islam Negeri Walisongo Semarang, Indonesia

Abstract

The Modified Left-Right Symmetry Model is an extension of the Standard Model. This model introduces left-handed neutrinos to the right sector and a doublet scalar field to the left sector. This model cannot yet explain the mass generation of fermions and neutrinos. This study is theoretical research using the literature review method. Generating the masses of fermions (quark up-down) and electrons through spontaneous symmetry breaking in Yukawa's Lagrangian term produces a particle mass in the left sector, the same as the calculations in the Standard Model. The masses of fermions (up-down quarks) and electrons for the right sector produced in this study are much more massive than those of fermions (up-down quarks) and the left sector. The neutrino masses produced in this study are by following the Seesaw Mechanism. That is, if one neutrino mass is massive, then the other neutrino masses will be light.

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1. Introduction

The theory of elementary particle interactions is still developing today. Scientists are trying to find a single theory that can unify the four elementary particle interactions, which include strong, weak, electromagnetic and gravitational interactions. However, until now, the theory can only unify the four particle interactions[1]. Currently, the Standard Model is a fairly well-established model for explaining weak, electromagnetic and strong interactions [2]. This model is built on the tera group $SU(3)_c \otimes SU(2)_l \otimes U(1)_Y$. According to physicists, gravitational interactions play a relatively weak role in interactions between elementary particles. Only two interactions dominate elementary particle interactions, namely weak and electromagnetic interactions.

The GWS model was initiated by Glashow, Weinberg and Salam[3]. This model combines weak and electromagnetic interactions called the electroweak theory[4][5]. The Standard Model has the advantage of predicting the
masses of the Tera W and Z bosons with the calculation results obtained by showing $M_W = 82 \pm 2 \text{ GeV}$ and $M_Z = 92 \pm 2 \text{ GeV}$ [6]. Not only that, in 2012 CERN announced an extraordinary discovery by finding the mass of the Higgs boson, which had been predicted in the Standard Model as the particle that causes fermions to have mass [2][7]. Although the Standard Model is considered the most established model, this model still has drawbacks, including being unable to explain the existence of dark matter, the phenomenon of neutrino oscillations and being unable to predict the mass of neutrinos [8][9]. This condition makes it necessary to expand the model to address these weaknesses.

New Left Right Symmetry expands the Standard Model by introducing a global quantum number of 1 for fermions, -1 for antifermions and 0 for their scalars [10][11]. Another model is Mirror Model. This model consists of Standard Model particles, right-handed neutrino and mirror particles which will be used as candidates for dark matter [12]. The next model is the Modified Left Right Symmetry Model. This model introduces the Standard Model particle with the addition of right-handed neutrinos $\nu_R$ in the left sector, a doublet scalar field $\Delta_L$ and a singlet scalar field $\eta$ and $\zeta$. Each particle in the left sector has a pair of particles in the right sector. This Modified Left Right Symmetry Model cannot explain the neutrino and fermion mass [13].

The Standard Model divides fermion particles into two types, namely quarks and leptons. Quarks are divided into up, down, charm, strange, top and bottom quarks. While leptons consist of electrons, electron neutrinos, muon neutrinos, and tauon neutrinos [1]. The mass of fermions can be generated by constructing of Yukawa’s Lagrangian terms, which are then carried out spontaneously, breaking the symmetry. This mechanism can only generate fermion masses (Up, Down and Electrons), not neutrino masses [14].

Neutrinos are one of the elementary particles of the lepton family. Pauli first introduced neutrinos in 1930 to explain beta decay [15]. Neutrinos are then considered hypothetical particles. It is because neutrinos have no charge and their mass is very small. In contrast to the Standard Model predictions, the Standard Model predicts that neutrinos are massless. It is because neutrinos only have one clarity, left handed [16][17]. The Seesaw mechanism is a mechanism that can be used to generate neutrino masses. This mechanism introduces right-handed neutrinos with a massive mass [18][19].

Based on the explanation above, the Modified Left-Right Symmetry Model cannot explain the mass of fermions (quark up-down) and electrons and has not explained the mass of neutrinos, so further research is needed on this model. Mass fermions (up-down quarks) and electrons can be generated through spontaneous symmetry breaking in Yukawa’s Lagrangian and Seesaw Mechanisms[20].

2. Experiments Procedure

This Study is theoretical research conducted using a literature review related to the fermion mass formulation in the Modified Left-Right Symmetry Model. The stages are shown in Figure 1. The stages carried out in this research are:

1. Up-down quark mass generation and electrons  
   a. Information gathering  
   Information was collected through literature studies in journals, books, and articles related to the Standard Model, Left-Right Symmetry, and the Seesaw Mechanism.

   b. Material depth  
   In-depth study of the material searched through literature studies, including the Standard Model, Left-Right Symmetry, and the Seesaw Mechanism.

   c. Construction of the tribes in Lagrangian Yukawa  
   Constructing the possible terms in Yukawa's Lagrangian is obtained by multiplying the fundamental representation of each particle and must results obtained (1,1,1,0).

   d. Up-down quark mass generation and electrons  
   Generation of Up, Down and Electron masses begins with grouping these
terms based on their particles, namely Up, Down and Electrons. The next step is to expand around the VEV value of the scalar field. The VEV value is substituted for each of these terms.

e. Fermion mass formulation
After the VEV value is substituted, the mass formulation for each Up, Down and Electron particle will be obtained.

b. Material depth
In-depth study of the material searched through literature studies, including the Standard Model, Left-Right Symmetry, and the Seesaw Mechanism.

c. Construction of the tribes in Lagrangian Yukawa
Constructing the possible terms in Yukawa's Lagrangian is obtained by multiplying the fundamental representation of each particle and must result in obtained (1,1,1,0).

d. Neutrino generation
Generating the neutrino mass begins with grouping the tribes that contain neutrinos at Lagrangian Yukawa, then generating their neutrino mass through the Seesaw mechanism, the steps of which are:

I. Look for the T matrix, a combination matrix of the Dirac neutrino mass and the mass term M.

II. Finding the self-valued T-Matrix with the equation \[ \det (T - \lambda I) \]

III. Using the 15-day Maple 2022 Trial software. Diagonalize the T Matrix.

IV. Neutrino mass formulation.

3. Result and Discussion
The Left Right Symmetry Model is an extension of the Standard Model, which currently has many variations[10][21]. One variation of the Left Right Symmetry Model is the Modified Left Right Symmetry Model. The modified Left Right Symmetry Model emerged as a response to the inability of the Standard Model to predict the mass of neutrinos. The particles contained in this model are the Standard Model particles in the left sector. There is a partner in the right sector and the addition of right-handed neutrinos \( \nu_R \), which will be used to generate neutrino masses through the Seesaw mechanism[22][23]. The list of particles in the Modified Left-Right Symmetry Model and their fundamental representation is shown in Table 1. The scalar field and fundamental representation
for the Modified Left-Right Symmetry Model are shown in Table 2.

<table>
<thead>
<tr>
<th>Left Sector</th>
<th>Representative</th>
<th>Right Sector</th>
<th>Representative</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell_L$</td>
<td>(1,2,1,-1)</td>
<td>$\ell_R$</td>
<td>(1,1,2,-1)</td>
</tr>
<tr>
<td>$\nu_R$</td>
<td>(1,1,1,0)</td>
<td>$N_R$</td>
<td>(1,1,1,0)</td>
</tr>
<tr>
<td>$e_R$</td>
<td>(1,1,1,-2)</td>
<td>$E_R$</td>
<td>(1,1,1,-2)</td>
</tr>
<tr>
<td>$q_L$</td>
<td>(3,2,1,$\frac{1}{3}$)</td>
<td>$Q_R$</td>
<td>(3,1,2,$\frac{1}{3}$)</td>
</tr>
<tr>
<td>$u_R$</td>
<td>(3,1,1,$\frac{4}{3}$)</td>
<td>$U_L$</td>
<td>(3,1,1,$\frac{4}{3}$)</td>
</tr>
<tr>
<td>$d_R$</td>
<td>(3,1,1,$\frac{1}{3}$)</td>
<td>$D_L$</td>
<td>(3,1,1,$\frac{2}{3}$)</td>
</tr>
</tbody>
</table>

The scalar field potential in the Modified Left-Right Symmetry Model that has been expanded around the VEV value is shown by Equation (1-6). 

$$\phi_L = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + h_{vL} \end{bmatrix}$$

$$\phi_R = \frac{1}{\sqrt{2}} \begin{bmatrix} v + h_{vR} \\ 0 \end{bmatrix}$$

$$\eta = h_\eta$$

$$\zeta = h_\zeta$$

$$\Delta_L = \frac{1}{\sqrt{2}} [h_{\Delta L}]$$

$$\Delta_R = \frac{1}{\sqrt{2}} [h_{\Delta R}]$$

VEV values for $v_L$ and $v_R$ is shown by Equation (7-8).

$$v_L = \pm \sqrt{\frac{\mu_1^2 - \frac{\epsilon_6 \mu_3^2}{2 \lambda_5}}{\lambda_1 - \frac{\epsilon_6}{4 \lambda_5}}}$$

$$v_R = \pm \sqrt{\frac{\mu_5^2 - \frac{\epsilon_6 \mu_2^2}{2 \lambda_4}}{\lambda_5 - \frac{\epsilon_6}{4 \lambda_4}}}$$

The fermion particles reviewed in this study include up quarks, down quarks, electrons and neutrinos for the left and right sectors. Generation of up, down quark, and electron masses is done by destroying the symmetry of Yukawa's Lagrangian terms, while generating neutrino masses is done through the Seesaw mechanism involving the mass term $M$ and the neutrino mass term $\mu$. 

Electron masses, up quarks down and neutrinos can be generated by constructing Yukawa's Lagrangians in both sectors of the Modified Left Right Symmetry Model. Yukawa's Lagrangian on the Modified Left-Right Symmetry Model is shown by Equation (9).

$$L_Y = -G_e (\ell_L \phi_L e_R + \ell_R \phi_R^c E_L)$$

In order to generate the fermion mass (up-down quarks and electrons) of the left and right sectors, the construction of Yukawa's Lagrangian term is part of Equation (9). The Yukawa Lagrangian terms used to generate electron masses in the left and right sectors are shown by Equation (10).

$$L_Y = -G_e (\ell_L \phi_L e_R + \ell_R \phi_R^c E_L)$$

Left and right sector electron masses are generated by substituting values $\phi_L$. 

Table 2 Modified Left-Right Symmetry Model Scalar Field

<table>
<thead>
<tr>
<th>Left Sector</th>
<th>Representative</th>
<th>Right Sector</th>
<th>Representative</th>
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<tbody>
<tr>
<td>$\phi_L$</td>
<td>(1,2,1,-1)</td>
<td>$\phi_R$</td>
<td>(1,2,1,-1)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>(1,1,1,0)</td>
<td>$\eta^*$</td>
<td>(1,1,1,0)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>(1,1,1,-2)</td>
<td>$\zeta^*$</td>
<td>(1,1,1,0)</td>
</tr>
<tr>
<td>$\Delta_L$</td>
<td>(1,2,1,$\frac{1}{3}$)</td>
<td>$\Delta_R$</td>
<td>(1,2,1,$\frac{1}{3}$)</td>
</tr>
</tbody>
</table>

Table 1 List of Particles and Fundamental Representatives

and $\phi^c_R$ whose definition is shown by Eq. (11-12).

$$\bar{\phi}_L^c = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -v_L + h_{UL} \\ v_L & 0 \end{bmatrix}$$  \hspace{1cm} (11)

$$\bar{\phi}_R^c = \frac{1}{\sqrt{2}} \begin{bmatrix} v_R + h_{UR} & 0 \\ 0 & 0 \end{bmatrix}$$  \hspace{1cm} (12)

Equation (11-12) is then substituted into Equation (10) so that the electron mass in the right sector ($m_{E_R}$) and the electron mass in the left sector ($m_e$) is obtained as shown by Equation (13-14).

$$m_{E} = -\frac{G_e}{\sqrt{2}} v_R$$  \hspace{1cm} (13)

$$m_{e} = -\frac{G_e}{\sqrt{2}} v_L$$  \hspace{1cm} (14)

The electron mass in the left sector shown by Equation (14) by following the electron mass calculated based on the Standard Model calculations [24]. By substituting the values $v_R$ and $v_L$ contained in Equation (7-8) and assuming that $v_R > v_L$ will be shown that will be known that the mass of electrons in the right sector is greater than the mass of electrons in the left sector ($m_{E} > m_{e}$).

The Yukawa Lagrangian terms are used to generate the quark-up masses in the left and right sectors, shown by Equation (15).

$$L_Y \supset -G_u \left( \bar{q}_L^c \phi_L^c u_R + \bar{Q}_R \phi_R^c D_L \right) + h.c$$  \hspace{1cm} (15)

It’s the same as generating the mass of electrons. Generating the up quark masses for the right and left sectors can also be done by substituting the values $\phi_R^c$ shown by Equation (2) and $\phi_L^c$ those shown by Equation (16).

$$\phi_L^c = \frac{1}{\sqrt{2}} \begin{bmatrix} v_L + h_{UL} & 0 \\ 0 & 0 \end{bmatrix}$$  \hspace{1cm} (16)

The next step to generate the quark up masses in the right and left sectors is to substitute Equation (2) and Equation (16) into Equation (15), in order to obtain the up quark masses for the right sector ($m_{u}$) and the up quark masses in the left sector ($m_{\bar{u}}$) shown by Eq. (17-18).

$$m_u = -\frac{G_u}{\sqrt{2}} v_R$$  \hspace{1cm} (17)

$$m_{\bar{u}} = -\frac{G_u}{\sqrt{2}} v_L$$  \hspace{1cm} (18)

The mass of the quark up in the left sector shown by Equation (18) results by following the mass of the quark up based on the calculation results of the Standard Model [25]. It is assumed that the value $v_R > v_L$ can be obtained that the mass of the quark up in the right sector is greater than the mass of the quark up in the left sector ($m_u > m_{\bar{u}}$).

The first step to generate the mass of the quark down for the right and left sectors is to determine the Yukawa Lagrangian for the down quark obtained from Equation (9). Yukawa’s lagging for down quarks is shown by Equation (19).

$$L_Y \supset -G_d \left( \bar{q}_L^c \phi_L^c d_R + \bar{Q}_R \phi_R^c D_L \right) + h.c$$  \hspace{1cm} (19)

The quark down masses for the right and left sectors can be generated by substituting the scalar field values $\phi_L^c$ and $\phi_R^c$ that is shown by Equation (11-12) to Equation (19) so that the quark down masses for the right ($m_{d}$) and left sectors ($m_{\bar{d}}$) will be obtained which is shown by Equation (20-21).

$$m_{d} = -\frac{G_d}{\sqrt{2}} v_R$$  \hspace{1cm} (20)

$$m_{\bar{d}} = -\frac{G_d}{\sqrt{2}} v_L$$  \hspace{1cm} (21)
The quark down mass for the left sector shown by Equation (21) is the same as the down quark mass obtained from the Standard Model calculations [25]. The value $v_R$ is assumed to be greater than $v_L$, the value ($v_R > v_L$), so it can be obtained that the mass of the down quark in the right sector is greater than the mass of the down quark in the left sector ($m_D > m_d$).

Neutrino mass is generated by identifying the mass term obtained from Yukawa's Lagrangian in Equation (9). The neutrino mass term is indicated by Equation (22).

$$L_Y = -G_\nu \left( \bar{\nu}_L \phi_L \nu_R + \bar{\nu}_R \phi_R N_L \right) + M \bar{\nu}_R N_L$$

(22)

The scalar field $\phi_L$ and $\phi_R$ that, which has been expanded around the vacuum expectation value is shown by Equations (23-24).

$$\phi_L = \frac{1}{\sqrt{2}} \begin{bmatrix} v_L + h_{vL} \\ 0 \end{bmatrix}$$

(23)

$$\phi_R = \frac{1}{\sqrt{2}} \begin{bmatrix} v_R + h_{vR} \\ 0 \end{bmatrix}$$

(24)

Lagrange in Equation (22) gives rise to a scalar field $\bar{\phi}_L^c$, whose definition is shown by Equation (25).

$$\bar{\phi}_L^c = \begin{bmatrix} v_L + h_{vL} \\ 0 \end{bmatrix}$$

(25)

The next step is to substitute Equation (24-25) into Equation (22) to obtain Equation (26).

$$L_Y = -G_\nu \frac{v_L (\bar{\nu}_L v_R + \bar{\nu}_R v_L)}{\sqrt{2}}$$

$$-G_\nu \frac{h_L (\bar{\nu}_L v_R + \bar{\nu}_R v_L)}{\sqrt{2}}$$

$$-G_\nu \frac{v_R (\bar{N}_R N_L + \bar{N}_L N_R)}{\sqrt{2}}$$

$$-G_\nu \frac{h_R (\bar{N}_R N_L + \bar{N}_L N_R)}{\sqrt{2}} - M \bar{\nu}_R N_L$$

(26)

Based on Equation (26) the neutrino mass term is shown by Equation (27).

$$L_Y \geq -G_\nu \frac{v_L (\bar{\nu}_L v_R + \bar{\nu}_R v_L)}{\sqrt{2}} - G_\nu \frac{v_R (\bar{N}_R N_L + \bar{N}_L N_R)}{\sqrt{2}} - M \bar{\nu}_R N_L$$

(27)

Equation (27) shows that the neutrino mass term consists of two types: the Dirac neutrino mass term and the M mass term. The neutrino mass can be generated through the Seesaw mechanism. The first step that can be taken to find the neutrino mass is to express Equation (27) in the matrix form shown in Equation (28).

$$L_Y \geq \frac{1}{\sqrt{2}} \begin{bmatrix} v_L & N_R & N_L & \bar{\nu}_L \\ 0 & 0 & 0 & -G_\nu v_L \\ 0 & -G_\nu v_R & 0 & -M \\ -G_\nu v_L & 0 & -M & 0 \end{bmatrix}$$

(28)

The neutrino mass term matrix from Equation (28) is shown by Equation (29).

$$\begin{bmatrix} v_L & N_R & N_L & \bar{\nu}_L \\ 0 & 0 & 0 & -G_\nu v_L \\ 0 & -G_\nu v_R & 0 & -M \\ -G_\nu v_L & 0 & -M & 0 \end{bmatrix}$$

(29)

The next step is to substitute $\frac{v_L}{v_R} \omega \ll 1 \frac{M}{B} \ll 1 $, and then the self-value and the swavector (U Matrix) for matrix $T$ are shown by Equation (30-31).
The diagonalized T matrix is
\[ \begin{align*}
&\frac{\sqrt{2}G_v^2 v_R^2 + 4M^2 + 2v_R G_v^2}{2} \\
&\frac{\sqrt{2}G_v^2 v_R^2 + 4M^2 + 2v_R G_v^2}{2} \\
&\frac{\sqrt{2}G_v^2 v_R^2 - 2v_R G_v^2}{2} \\
&\frac{\sqrt{2}G_v^2 v_R^2 - 2v_R G_v^2}{2}
\end{align*} \]
\[ \begin{align*}
&= \left[ \begin{array}{cccc}
\frac{\sqrt{2}G_v^2 v_R^2 + 4M^2 + 2v_R G_v^2}{2} & 0 & 0 & 0 \\
0 & \frac{\sqrt{2}G_v^2 v_R^2 + 4M^2 + 2v_R G_v^2}{2} & \frac{4MG_v^2 v_L^2}{\sqrt{2}PQ} & 0 \\
0 & \frac{4MG_v^2 v_L^2}{\sqrt{2}PQ} & \frac{\sqrt{2}G_v^2 v_R^2 - 2v_R G_v^2}{2} & \frac{4MG_v^2 v_L^2}{\sqrt{2}PQ} \\
0 & 0 & -\frac{4MG_v^2 v_L^2}{\sqrt{2}PQ} & \frac{\sqrt{2}G_v^2 v_R^2 - 2v_R G_v^2}{2}
\end{array} \right]
\end{align*} \]

Diagonalization of the T Matrix is the next step that must be done. The diagonalization of the T matrix is carried out with the condition \( U^T U \).

The diagonalized T matrix is shown in Eq. 34.

\[ \begin{align*}
&= \left[ \begin{array}{cccc}
\frac{\sqrt{2}G_v^2 v_R^2 + 4M^2 + 2v_R G_v^2}{2} & 0 & 0 & 0 \\
0 & \frac{\sqrt{2}G_v^2 v_R^2 + 4M^2 + 2v_R G_v^2}{2} & \frac{4MG_v^2 v_L^2}{\sqrt{2}PQ} & 0 \\
0 & \frac{4MG_v^2 v_L^2}{\sqrt{2}PQ} & \frac{\sqrt{2}G_v^2 v_R^2 - 2v_R G_v^2}{2} & \frac{4MG_v^2 v_L^2}{\sqrt{2}PQ} \\
0 & 0 & -\frac{4MG_v^2 v_L^2}{\sqrt{2}PQ} & \frac{\sqrt{2}G_v^2 v_R^2 - 2v_R G_v^2}{2}
\end{array} \right]
\end{align*} \]

The neutrino mass resulting from Equation (34) is shown by Equation (35-38).

\[ m_{\nu} \] 
\[ m'_{\nu} \] 
\[ m_N \] 
\[ m'_N \]

Equation (35-38) shows that the neutrino mass \( m_{\nu} \) is very massive compared to the neutrino mass \( m'_{\nu} \). It also applies to neutrino masses \( m_N \) with neutrino masses \( m'_N \), which are massive neutrino masses, while light neutrino masses are \( m'_N \). Conditions like this show a connection with the Seesaw Mechanism, namely when \( m_1 \) (neutrons) has a massive mass, then \( m_2 \) (neutrinos) will have a light mass. The neutrino mass produced in this study differs from the neutrino mass produced in research conducted by Adam et al. [10] in 2020.

4. Conclusions

The masses of the up-down quark fermions and electrons in the left sector are

\[ m_u = \frac{-g_u}{\sqrt{2}} v_L \]  
\[ m_d = \frac{-g_d}{\sqrt{2}} v_L \]  
\[ m_e = \frac{-g_e}{\sqrt{2}} v_L \]

respectively. The masses of up-down quark fermions and electrons in the right sector are

\[ m_U = \frac{-g_u}{\sqrt{2}} v_R \]  
\[ m_D = \frac{-g_d}{\sqrt{2}} v_R \]  
\[ m_E = \frac{-g_e}{\sqrt{2}} v_R \]

respectively. Assuming \( v_R \gg v_L \), the fermion mass for the right sector is greater than the left sector fermion mass. The mass of the neutrino produced by the Seesaw Mechanism is

\[ m_{\nu} = \frac{4G_v^2 v_L^2 M}{G_v^2 (v_R^2 - v_R)} \]  
\[ m'_{\nu} = -\frac{4G_v^2 v_L^2 M}{G_v^2 (v_R^2 - v_R)} \]  
\[ m_N = -\frac{16M^3 G_v^2 v_L^4}{G_v^2 v_R^4 + v_R^2} \]  
\[ m'_N = -\frac{16M^3 G_v^2 v_L^4}{G_v^2 v_R^4 + v_R^2} \]
Acknowledgements

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References