Phenomenon, 2025, Vol. 15 (No. 1), pp. 173-197
Phenomenon : Jurnal Pendidikan MIPA
phenomenon@walisongo.ac.id

Metacognitive Process of Mathematics Education Students in Solving Calculus Problems on Integral Material

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Abstract

One of the essential skills for learning mathematics and solving mathematical problems is metacognition—the ability to control and evaluate one's own thinking. Key aspects of metacognition include awareness/planning, monitoring, and evaluation. This study focused on mathematics education students enrolled in an integral calculus course. It aimed to assess the students' metacognitive abilities and strategies when solving integral calculus problems involving area, the volume of solids of revolution, and integration techniques. The study employed a case study design and a descriptive-qualitative methodology. Three students of the same gender served as the research subjects. The findings indicate that students who demonstrated strong planning, monitoring, and evaluation skills—and who employed a wider range of metacognitive strategies—were able to solve more mathematical problems than those with weaker metacognitive skills who failed to utilize such strategies effectively. A positive correlation was observed between the number of metacognitive strategies employed and problem-solving proficiency. Evaluation was the metacognitive aspect most lacking among the students, as they frequently failed to verify their final answers.

Keywords: Metacognitive, Problem solving, Calculus integral

Proses Metakognitif Mahasiswa Pendidikan Matematika dalam Memecahkan Masalah Kalkulus Materi Integral

Abstrak

Salah satu kemampuan yang harus dimiliki seseorang jika ingin mempelajari matematika dan memecahkan masalah matematika adalah kemampuan metakognitif, yaitu kemampuan untuk mengendalikan dan mengevaluasi pemikiran sendiri. Aspek dari metakognitif meliputi kesadaran/perencanaan, monitoring dan evaluation. Penelitian ini dilakukan pada mahasiswa jurusan pendidikan matematika yang mengambil mata kuliah kalkulus integral. Penelitian ini dilaksanakan untuk mengetahui pencapaian kemampuan metakognitif dan strategi metakognitif mahasiswa dalam memecahkan masalah kalkulus integral yang meliputi luas daerah, volum benda putar dan teknik pengintegralan. Penelitian ini termasuk penelitian dengan desain studi kasus. Metode yang digunakan dalam penelitian ini adalah deskriptif kualitatif. Subjek penelitian diambil dari jenis kelamin yang sama. Subyek penelitian berjumlah tiga orang mahasiswa. Hasil penelitian menunjukkan

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ISSN: 2088-7868, e-ISSN 2502–5708

bahwa mahasiswa yang mempunyai kemampuan perencanaan, monitoring dan evaluasi yang baik dan menggunakan lebih banyak strategi metakognitif dapat memecahkan lebih banyak masalah matematika dibandingkan mahasiswa yang memiliki kemampuan metakognitif kurang dan tidak menggunakan strategi metakognitif dengan baik. Semakin banyak strategi metakognitif yang digunakan oleh mahasiswa, semakin baik kemampuan pemecahan masalahnya. Aspek metakognisi yang kurang dimiliki oleh mahasiswa adalah aspek evaluasi karena mahasiswa sering tidak melakukan pengecekan kembali terhadap jawaban yang diselesaikannya.

Kata kunci: Metakognisi, Pemecahan masalah, Kalkulus integral

INTRODUCTION

Calculus is a mathematics course within the analysis group. Integral calculus courses discuss integrals and their applications, along with integration techniques. Based on the author's experience as an integral calculus instructor, students encounter several difficulties in solving integral problems, including difficulty understanding the problem, difficulty drawing and analyzing graphs, difficulty determining integral limits if the limits are not specified, difficulty determining integral problem-solving methods, and difficulty performing integral calculations. Interviews revealed that these difficulties are caused by several factors, including a lack of practice with integral problems and a lack of understanding of the concept of derivatives, a prerequisite for integrals (Monariska, 2019). Difficulties in learning integrals are also caused by students' weaknesses in using problem-solving strategies (Hashemi et al., 2015).

In general, if someone wants to learn mathematics and want to solve mathematical problems, there are several abilities that they must have, including conceptual understanding, mathematical algorithm skills, mathematical process skills, positive attitudes toward mathematics, and metacognitive abilities (Zubaidah, 2017). Conceptual understanding will help someone choose the right design/solution procedure (Supriadi et al., 2021). Mathematical algorithm skills will help someone solve problems in a sequential, step-by-step manner. According to (Arslan et al., 2012), a positive attitude toward mathematics can improve mathematics grades. Metacognition can help someone in learning mathematics and solving mathematical problems, because metacognition is an individual's awareness/control of their own thinking, evaluation, and regulation of their own thinking (Wilson & Clarke, 2004). A student's failure or success is caused by the use of their metacognition (Charles & Silver, 1988). Therefore, mathematical

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ISSN: 2088-7868, e-ISSN 2502–5708

difficulties can be experienced by students who do not use their cognitive and metacognitive strategies.

Learning can be enhanced through metacognitive monitoring (Paris & Winograd, 2013). Metacognition is a person's awareness or belief in monitoring their own thought processes to improve their strategies and correct mistakes, thus optimizing learning (Novita et al., 2018). This can be explained by the fact that when students experience metacognition, they will be able to control their learning process, starting from planning learning, choosing the right strategy, and monitoring their learning progress. If errors and deficiencies are found, they will be able to correct them and make these habits their learning strategies. According to Butler and Winne in (Du Toit & Kotze, 2009), the most effective way to learn is through self-regulation. The meaning of self-regulation is how students can ask themselves about the number of tasks they have done and then observe how well they have done the work. According to (Camahalan, 2006) in their research, student learning achievement will increase if students are able to control themselves and can apply metacognitive strategies.

Flavel in (Du Toit & Kotze, 2009) defines metacognitive strategies as consciously monitoring a student's cognitive to achieve certain goals, for example when students ask themselves about the tasks they have completed, how well they answered the questions. Furthermore, Toit and Kotze developed metacognitive strategies in 13 variations of strategies, namely Planning strategy, Generating questions, Choosing consciously, Setting and pursuing goals, Evaluating the way of thinking and acting, Identifying the difficulty, Paraphrasing, elaborating and reflecting learners' ideas, Clarifying learners' terminology, Problem-solving activities, Thinking aloud, Journal-keeping, Cooperative learning, Modeling.

Previous research related to the difficulty of solving integral calculus problems was conducted by (Sertin Allolayuk1, Delfince Tjenemundan2, 2023) with research results in the form of difficulties in solving integral calculus problems including difficulty drawing graphs, difficulty determining integral formulas and difficulty solving integrals. In addition, the results of other studies revealed that their metacognitive knowledge was more dominant in declarative knowledge than procedural knowledge in solving integral problems (Yuliana M., 2022). Based on this, the researcher conducted research that focused more on the achievement of metacognitive skills and the use of

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ISSN: 2088-7868, e-ISSN 2502–5708

metacognitive strategies in solving integral calculus problems. In this article will discuss 1) the extent of the use of students' metacognitive skills in solving integral calculus problems, 2) how the use of metacognitive strategies used by students in learning integral calculus.

The term metacognition was first introduced by John Flavell in 1979 (Erickson, 2015). Metacognition is often interpreted as "thinking about thinking." Pate and Miller in (Irham, 2015) state that metacognition is the awareness and ability to regulate and control one's thought processes. According to Flavell, cognition and metacognition are indistinguishable. Cognition is all activities and processes in which things received by the five senses are transformed, interpreted, stored, and retrieved for use (Bayne et al., 2019). Metacognition is an individual's awareness and ability to monitor their thought processes. Examples of cognitive processes are reading, calculating, and doing assignments. Examples of metacognitive activities include how to calculate, choosing how to work on a problem, choosing which formula to use, managing time to work on a problem, and so on. From this, it can be concluded that the function of metacognition is to control one's cognition or thinking.

Flavell's metacognitive model consists of knowledge, experience, goals (tasks), and actions (strategies). Metacognitive knowledge refers to explicit experiences of one's own cognitive abilities. Metacognitive awareness refers to the feelings and experiences experienced while engaging in cognitive processes. Cognitive strategies are processes that control cognitive activities to achieve cognitive goals and monitor the success of those goals. Metacognitive control is a decision made based on the results of the monitoring process. Metacognitive strategies will support students in improving their academic achievement (Valencia-Vallejo et al., 2019).

The components of metacognition consist of metacognitive knowledge and metacognitive experience (Schraw, 1998). The cognitive knowledge component includes self-knowledge, task knowledge, and strategy knowledge. Metacognitive knowledge directs cognitive search for stored memories, both consciously and automatically. The metacognitive experience component relates to metacognitive strategies. These strategies function to control cognitive activity to achieve goals and monitor the success of those cognitive goals.

Decision-making can be controlled by metacognition. This is evident in a person's

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behavior or strategies when faced with a problem. For example, if a student is required to take an exam but feels unprepared, the student may postpone the exam to prepare first. This decision can be made to achieve optimal exam results. This metacognitive control process can influence human behavior and cognition, so this control process should always be honed/improved to help decision making in any situation (Erickson, 2015).

These two metacognitive components are essential in learning activities. With metacognitive components, students will be able to monitor everything they know to solve a problem. With good metacognition, students can organize and monitor their own learning, manage their study time appropriately, use appropriate learning strategies and methods suited to their abilities, and use their cognition to devise formulas or algorithms appropriate to the problem. The expected end result is the achievement of learning objectives and the ability to monitor their own learning progress.

Metacognitive strategies are strategies or steps that train someone to habitually think about their thought processes. They refer to the conscious monitoring of one's cognitive strategies to achieve specific goals, the sequence of processes to control learning, and are considered higher-order thinking skills. Metacognitive strategies are a way for someone to manage their own cognitive activities, particularly in learning and thinking. The application of metacognitive strategies has been shown to improve cognitive abilities and skills, control memory, and enhance self-confidence, leading to independent and meaningful learning (Mitsea & Drigas, 2019). Applying metacognitive strategies to children from an early age offers numerous benefits, including improving learning outcomes and fostering independent learning. It is hoped that by habituating metacognitive strategies, individuals will be able to assess their limitations and know what to do when faced with a problem.

The benefits of metacognitive strategies for an individual are immense. Educators agree that the more metacognitive strategies a person learns for following, encoding, storing, transferring, and solving problems, the more they will become self-learners and independent thinkers. An educator should be able to provide effective teaching to their students. Good teaching includes helping students understand how to learn, remember, think, and motivate themselves. This demonstrates that teaching students how to learn is one of the primary goals of education. Therefore, teaching learning strategies (cognitive strategies) is crucial and essential (Ikram & Aziz, 2017).

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Metacognition relates to thinking about one's own thoughts. Metacognitive strategies will help a person before, during, and after the learning process. Students can ask themselves questions that encompass the three components of metacognitive skills: planning, monitoring, and evaluating. Some self-questioning activities that a learner can engage in while studying or solving math problems include:

Table 1. metacognitive activities

Component	Learner Activity
Planning	1. What knowledge can help me solve this problem? 2. What should I do first to solve this problem 3. What is the relationship between what is known and what is asked? 4. What formula will I use to solve this problem
Monitoring	1. How do I solve this problem? 2. After this step, what next step should I take? 3. What other information do I need? 4. Is there another, easier way? 5. If there is a difficulty like this, how do you overcome it?
Evaluation	1. Are the steps I have taken correct? 2. Is there any other way to solve this problem? 3. Can this method be applied to other problems? 4. What are my weaknesses when facing this problem? 5. How can I overcome my weaknesses in this problem?

Metacognition plays a crucial role in communication, reading comprehension, writing, attention, memory, and problem-solving (Irham, 2015). Mathematics requires logic and reasoning to solve problems, while metacognition relates to the ability to control one's thought processes and use one's awareness to solve problems. This suggests a link between mathematics and metacognition. Metacognition directs or finds ways to solve problems by controlling one's thought processes. This is consistent with research conducted by Yildirim & Ersözlü (2013), which found a significant relationship between metacognition and mathematical problem-solving. This link, among other things, stems from the need to develop metacognitive tools to organize and manage the problem-solving process (Hancock & Karakok, 2021). Metacognition is a good predictor of problem-solving ability (Aurah et al. 2011). If someone wants to solve

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a problem, that person needs metacognition which will help that person understand and realize what problem needs to be solved and what efforts need to be made to get a solution (Kuzle, 2013).

An educator must possess skills that can support students' metacognitive processes, including self-directed learning skills, beliefs about self-directed learning and metacognition, and the ability to encourage self-directed learning and metacognition (Yes Karlen, Carmen, Nadja Hirt, Johannes Jud, Amina Rosenthal, Tabea, Daria Eberli, 2023). A person's metacognitive skills can be honed if the person is accustomed to solving problems, because when someone carries out problem-solving activities, the person will use higher-order thinking skills (Zoller & Pushkin, 2007), including critical and creative thinking. Critical and creative thinking skills must be supported by good metacognitive skills. Thus, metacognitive skills can support the improvement of higher-order thinking, thereby improving problem-solving abilities (Güner and Erbay 2021). Therefore, the purpose of this study is to describe the metacognitive abilities possessed by Mathematics Education students in solving integral problems.

RESEARCH METHODS

This research uses a case study design. A case study is a type of qualitative research in which the researcher conducts an in-depth exploration of a program, event, process, or activity involving one or more individuals. The case study examines the extent to which students achieve metacognitive skills through the use of metacognitive strategies in solving integral problems.

This research was conducted among 30 second-semester students at UIN Walisongo in 2024, taking integral calculus. The researcher administered integral test questions (integral of functions, area under a curve, and volume of solids of revolution) to assess their metacognitive skills and problem-solving abilities. The test instrument was empirically validated beforehand. The researcher also administered a questionnaire to determine the strategies used in solving integral problems. In this case study, the researcher selected three students as participants, with the criteria being: one with the highest, one with the average, and one with the lowest score.

Participant biographies

Gilang Gilang is a second-semester mathematics student who has loved mathematics since high

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(S1)	school. Galang has a relaxed character but can focus on the material even though he likes to sit at the back in his classroom, rarely asks questions in class, has a high curiosity always wants to try new things, and enjoys exploring his abilities in mathematics by utilizing YouTube and social media. Other
Adnan (S2)	Adnan is an ambitious mathematics student who can master mathematical material, in this case, calculus, has a high enthusiasm for learning, is enthusiastic, always sits at the front when studying in class, often asks questions if he doesn't understand the material, always tries to be the best.
Rama (S3)	Rama is a second-semester mathematics student who has liked mathematics since high school. Rama has a quiet character, doesn't socialize with his friends, prefers to sit at the back, and rarely asks questions in class but is diligent in doing the assignments given by the teacher.

In addition to considering test results, the researchers also recruited participants of the same gender to ensure the results were not influenced by gender. They also used pseudonyms for the participants. During the preparation stage, the researchers developed 13 indicators of metacognitive strategies formulated by Du Toit & Kotze (2009). These indicators include planning strategy, generating questions, choosing consciously, evaluating the way of thinking and acting, identifying the difficulty, paraphrasing, elaborating and reflecting learner ideas, clarifying learner terminology, problem-solving activities, thinking aloud, journaling, cooperative learning, and modeling. The researchers also developed a metacognitive strategy questionnaire consisting of 30 questions that led to the elaboration of the metacognitive strategy steps outlined by Toit and Kotze. Open-ended questions were designed by the researchers to validate the data through interviews with participants. Furthermore, the authors developed an integral material instrument consisting of four validated questions to examine the metacognitive processes involved in problem-solving.

In the implementation phase, the researcher administered four essay questions

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on integral material, covering simple function integrals and applications of integrals to determine the area and volume of rotating objects. The questions were given to 30 students. They completed the questions within 90 minutes. This test was conducted to determine the metacognitive processes involved in problem-solving, including their metacognitive skills and strategies. The next step was to select three participants with the highest, middle, and lowest scores: Gilang (S1), Adnan (S2), and Rama (S3). The next step was to administer a questionnaire about metacognitive strategies to the three participants. This questionnaire was administered with the aim of determining the metacognitive strategies frequently used by the two participants in solving integral problems. The next step was to interview the three participants to determine the validity of the data obtained during the essay test and when completing the questionnaire. The interviews were also conducted to further explore their metacognitive skills and the metacognitive strategies they chose.

RESULTS AND DISCUSSION

This section will discuss the metacognitive processes experienced by mathematics education students in solving integral problems. This exploration encompasses metacognitive skills, including planning, monitoring, and evaluation, as well as the strategies employed by the students. It also includes metacognitive knowledge, consisting of declarative knowledge, procedural knowledge, and conditional knowledge.

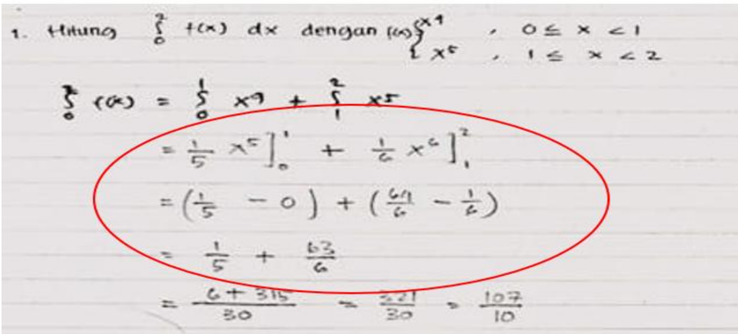

Image 1. Results of S1 Participants' Work on Question Number 1

Based on the image 1, it can be seen that S1 can do question number 1 perfectly. His preparation was very good, he added the integrals of each function. S1 has an awareness of the steps that must be taken to answer the existing problem. Planning activities arise when S1 remembers having received a question like this and having

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worked on a question like this. In the planning stage, participant S1 is aware of: (1) the material being faced, he is aware that he is facing material about integrals of piecewise functions, as seen from the following steps to solve them piecewise according to their domain, (2) the direction and purpose of the question given, he is aware that the question is to determine the integral value, (3) aware of what is known and what is asked, that what is known is a piecewise function with its respective domain and what is asked is the definite integral value for the function, as seen from his answering the question according to what is known and asked, (4) aware of the steps to solve that must be done, namely finding the indefinite integral for each function in its domain, (5) aware of what formulas are needed, namely the indefinite integral formula and the definite integral formula. This awareness can lead to problem-solving plans, expressed in the form of monitoring or regulation. Good planning is a crucial aspect of problem-solving (Mulyono & Hadiyanti, 2018). Regulatory or monitoring activities emerge when S1 creates a plan to solve the problem. The plan includes finding the definite integral value of each part according to its domain. He goes through each step of the solution in a well-organized and precise manner. He always rechecks or evaluates the calculations or solution methods used at each step, ensuring complete confidence in the accuracy of his calculations and solution methods.

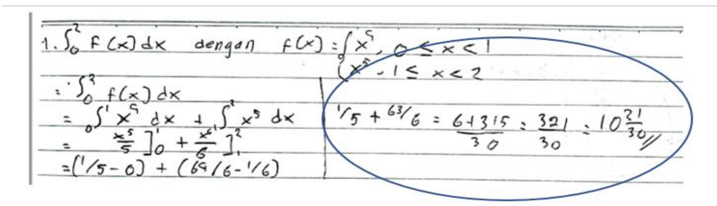

Image 2. S2 Answer Results for Question Number 1

Based on the image 2, it can be seen that S2 can do question number 1 perfectly. His preparation was very good, he added the integrals of each function. S2 has an awareness of the steps that must be taken to answer the existing problem. His habit of learning by exploring various tutorials on online media such as YouTube and Google has trained him to face various types of different questions. S2 is very familiar with the steps to solve the problem. His planning activity appeared when S2 remembered having received a question like this and had worked on a question like this. In the planning stage, participant S2 was aware of: (1) the material being faced, he was aware that he was facing material about integrals of piecewise functions. (2) the direction and purpose

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of the given problem, he realized that the problem was to determine the value of a definite integral (3) what is known and what is asked, that what is known is a piecewise function with its respective domain and what is asked is the definite integral value for the function (4) the steps for solving that must be done, namely finding the indefinite integral for each function in its domain, (5) what formulas are needed, namely the indefinite integral formula and the definite integral formula. Regulation/monitoring activities appeared when S2 solved the problem. The actions he took included finding the definite integral value piecewise according to its domain, carrying out calculation techniques correctly and carefully. Rechecking or evaluation in the calculations and methods that had been used was sometimes done by S2, but sometimes he did not do it if he was already confident and the problem he was facing was relatively simple.

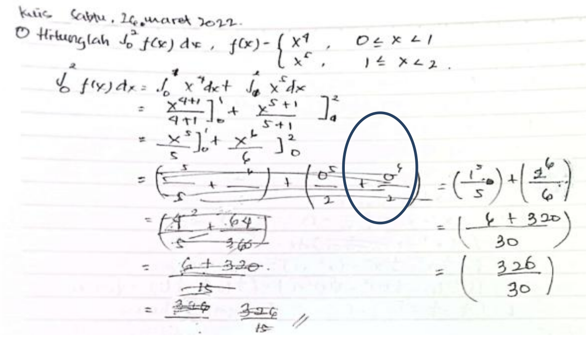

Image 3. S3 answer results for question number 1

Based on image 3, S3's answer to question 1 was incorrect when determining the limit of the piecewise function. The correct limit for the second function is from one to two, but during the answering process it was written as zero to two. During the planning stage, S3 realized that he was facing a problem about definite integrals so he worked on the problem by integrating the piecewise function according to its domain, the calculation technique and solution method used by S3 were correct but S3 did not write the integral limit correctly. In this case, S3 was unable to use the information correctly which resulted in an error in solving the problem. So it can be said that the planning carried out by S3 was good but the monitoring/regulation was not good. S3 also did not evaluate or recheck his answers.

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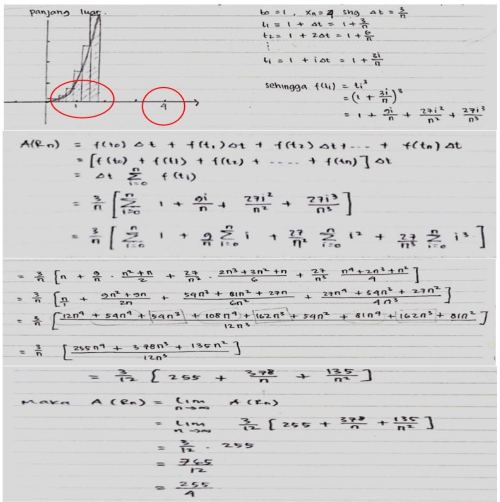

Image 4. S1 Answer Results for question number 2

Based on image 4, participant S1 completed question number 2 coherently and correctly. S1 realized that he had to solve a problem of the area under the curve using a rectangular approach. The planning he did was to describe a known function, and then partition the area under the function into n - partitions of the same length, thus forming an outer rectangle of n - pieces, find the length of the partition, that is $\Delta t = \frac{3}{n}$, so $t_i = \left(1 + \frac{3i}{n}\right)$ obtained. Then add up all the areas of the rectangle that have long sides $f\left(t_i\right)$ and the broad side Δt using the formula $A\left(R_n\right) = \Delta t \sum_{i=1}^{n} f\left(t_i\right)$ and by taking the limit for $n \to \infty$. Even though Participant S1's answer is correct, it has a drawback, namely that when drawing the function of the outer rectangle it does not start with $x_0 = 1$ and end with $x_n = 4$

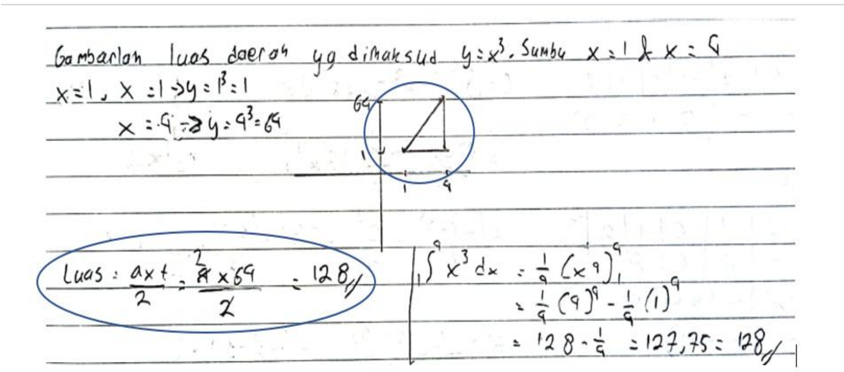

Image 5. S2 Answer Results for Question Number 2

Based on image 5, the results of S2's answer to question number 2, it can be seen that S2 has made an error, namely an error when drawing the function from to which he depicted as a triangle. S2 does not have good planning such as when drawing the graph of the function wrong, does not know the relationship between what is known and what is asked, there is an answer that looks for the area of a triangle even though there is no connection between the question and the area of a triangle. S2 is also unable to determine the area under the curve using the outer rectangular area approach. What S2 did was find the definite integral value using the second fundamental theorem of calculus, and even that result was wrong. So the concept he used to calculate the area under the curve is incorrect.

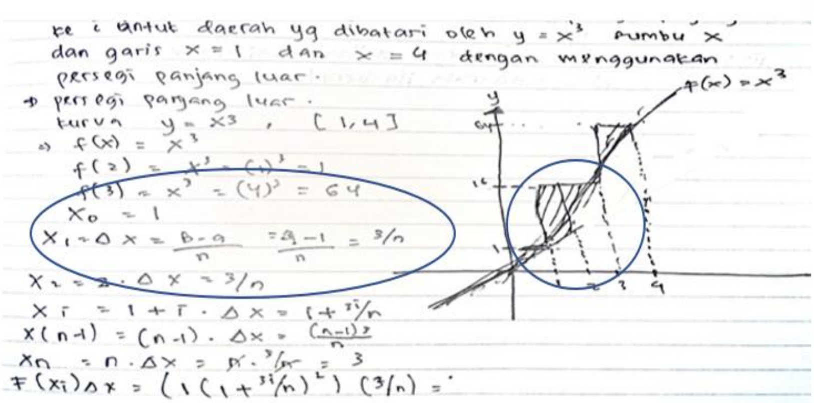

Image 6. S3 Answer Results for Question Number 2

Based on Image 6, the results of S3's answer to question number 2, S3 cannot solve this problem. The function $f(x)=x^3$ is not a linear, he drew the function using only two points, namely at $f(1)=1$ and $f(4)=64$ which are usually used to draw graphs of linear functions, so the graphic image and the length of the outer rectangle are wrong. When this was confirmed to S3, he said that he could not remember this material only a few things when it was taught in class. From this, it can be noted that S3 lacks declarative knowledge in the form of knowledge about facts, concepts, and other factors that influence his thinking in solving problems. The procedural knowledge in the form of stages/steps of completion is only mastered in the initial steps of completion, namely in the form of drawing a rectangle with boundaries $x=1$ up to $x=4$ then determining $\Delta x=\frac{3}{n}$ and entering the formula for the area of a rectangle, that is $f(x_i)\Delta x$, but not until completion because the ability to think only stops there. In this case, S3 still

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remembers the steps to solve it but forgets the formula used, in other words, his explicit memory fades more easily than his implicit memory (Schacter 1992). In terms of metacognitive activities for planning/metacognitive awareness, they have emerged but cannot be developed optimally, for monitoring activities they are not carried out well because they do not have metacognitive knowledge in the form of mastery of concepts and strategies for determining the area under the curve.

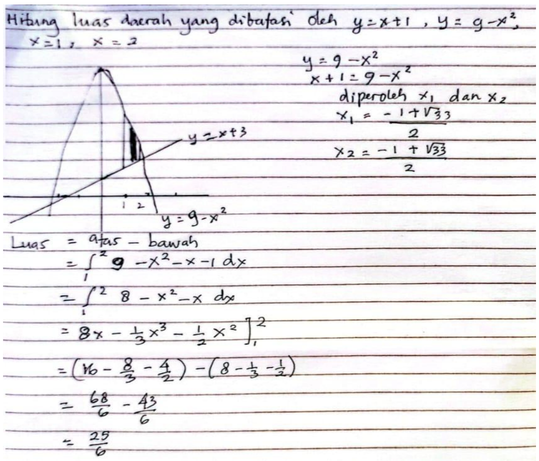

Image 7. S1 Answer to Question Number 3

Based on image 7, participant S1 completed question number 3 correctly. The planning he did was to draw a graph of the function $y = x + 1,$ and $y = 9 - x^2$ with limits $x = 1, x = 2$, then analyze the image until he obtained that the area is a definite integral of the function above minus the function below with predetermined limits, that is $\int_1^2 \left(9 - x^2\right) - \left(x + 1\right) dx$, the calculation technique he carried out was also correct. His cognitive monitoring process worked well. He knew every step of the solution, and the relationship between what was known and what was asked, and the formula he used was correct. Evaluation is always carried out by S1 by reviewing each stage of completion to ensure that there are no errors in the calculation methods and techniques. S1 participants already understand the integral application material for calculating the area of an area, so that their metacognition activities can run well.

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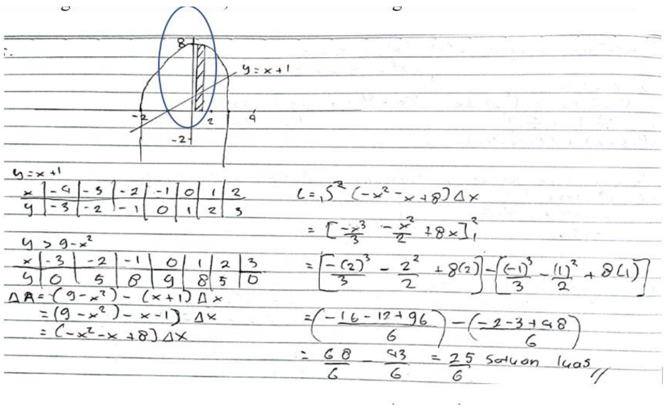

Image.8 S2 answer to question number 3

Based on image 8, Participant S2 did question number 3 correctly even though several steps did not need to be taken, for example, to draw a straight line only two coordinate points were needed but S2 used many unnecessary coordinate points. Apart from that, the shaded image where the area will be searched is also not correct, but in general, the planning carried out by S2 to solve the area problem is good and proper. When S2 confirmed the error, he said that the mistake was because S2 was not careful enough, he only remembered that the area between the 2 formula graphs was the graph above minus the graph below. The interesting thing that S2 did in question number 3 was that S2 could not describe the area he was looking for correctly, but S2 could find the area correctly. Participant S2 worked on this problem using the technique or principle of the function above minus the function below $L = \int_{1}^{2} (9 - x^2) - (x+1)dx$, that's all he remembered when the lecturer explained it in class. So procedural knowledge from S3 can help in solving problems regarding area area. Declarative knowledge is part of procedural knowledge (Ten Berge and Van Hezewijk 1999). So if someone's procedural knowledge is well developed then their declarative knowledge will adjust because for example someone already knows how to solve a problem at each step, they will definitely look for related theories to solve those steps. Thus, metacognitive knowledge from S2 has guided the monitoring and evaluation process of S2 well.

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ISSN: 2088-7868, e-ISSN 2502–5708

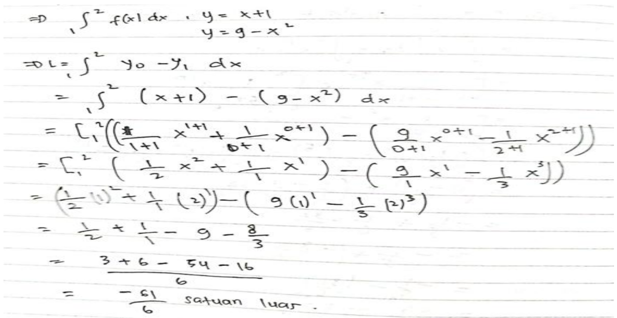

Image 9. S3 answer to question number 3

Based on image 9, the answer given by S3 to question number 3 is wrong. Participant S3 did not draw a graph of the function $y = 9 - x^2$ and $y = x + 1$ on a Cartesian plane whose area he wanted to find, this resulted in S3 not being able to carry out an analysis to determine the method he would use to find the area of the area in question correctly. When confirmed to S3 he said that S3 cannot draw graphs of quadratic and linear functions. At the planning stage, S3 already knew what he was going to do, but in terms of knowledge and problem solving procedures, S3 was unable to carry out the planning stages well, namely carrying out analysis of function graphs because S3 was unable to draw function graphs and planned to draw function graphs to analyze the solution. The result of S3's inability to draw function graphs was an error in determining the form/formula used to calculate the area under the curve, which should have been the case $\int_1^2 \left[\left(9 - x^2 \right) - \left(x + 1 \right) \right] dx$ but S3 wrote the formula $\int_1^2 \left[\left(x + 1 \right) - \left(9 - x^2 \right) \right] dx$. This happens because there is no synchronization in his cognition between his declarative knowledge and his planning. Apart from that, there was a conceptual error when inputting the integral limit, where S3 did not use the concept of definite integral $\int_a^b f(x)dx = F(b) - F(a)$. When confirmed to S3 he said that he knew the formula but applied it incorrectly in solving the problem. So in this case the process of monitoring and evaluating the implementation of the plan did not go well due to the weak declarative knowledge possessed by S3.

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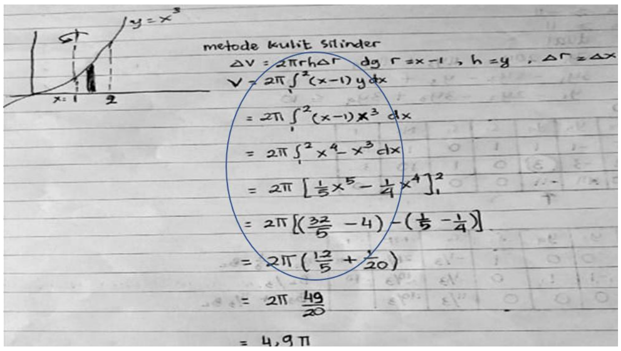

Image 10. S1 answer to question number 4

Based on image 10, the results of S1's answer to question number 4, it can be seen that S1 has done the question well. Participant S1 finds the volume of a rotating object by drawing a function graph $y = x^3$ with limits $x = 1$ up to $x = 4$ and rotated around a line $x = 1$ as the axis of rotation. Participant S1 carried out an analysis to find the volume of a rotating object using an upright rectangular cut to form a tube shell, so S3 used the tube shell method. S1 realized that the formula for the volume of a tube shell is $\Delta V = 2\pi rh\Delta r$ that the radius formed is $r = x - 1$, the height of the tube shell is $h = y = x^3$, and $\Delta r = \Delta x$. Participant S1 understands the material regarding the volume of rotating objects, the plan/strategy he used was correct so the results obtained were also correct. So metacognitive knowledge which includes declarative knowledge in the form of mastery of material, concepts, and solving strategies, procedural knowledge in the form of stages of problem-solving, and conditional knowledge in the form of situations in which an approach is used can be carried out correctly. So S1's metacognitive activities run well.

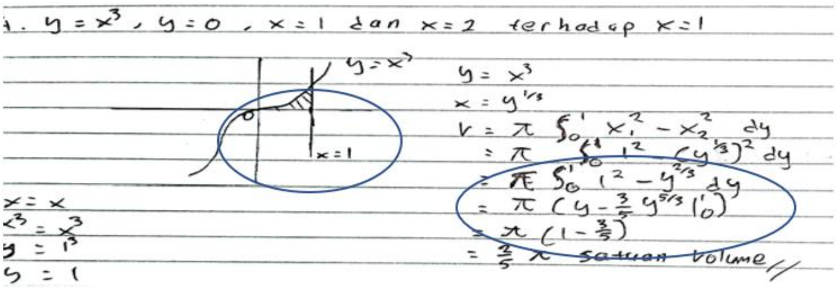

Image 11. S2 answer to question number 4

Based on image 11, participant S2 could not solve question number 4 correctly. Participant S2 could not describe the problem, namely that the boundary of the area to be rotated was $x = 1$ up to $x = 2$. Apart from that, S2 also did not make a rectangular slice that would be rotated around the line $x = 1$ so he had difficulty analyzing the shape

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of the rotating object so he could not determine the correct method for calculating the volume of the rotating object. When confirmed to S2 he said that the limit is $y = 0$ so that when $x = 1$ then $y = 1$ will be the upper limit. From the results of the solution and S2's statement, it can be said that S2 did not plan the solution well, that is, he was not careful in determining the integral limits so an error occurred in determining the integral limits. S2 is unable to determine the area to be rotated, this occurs because the specified integral limits are incorrect. So S2 is unable to use the data/information provided, and S2 experiences confusion when given quite a lot of integral limits. The process of monitoring what was to be done experienced obstacles because mastery of the material/concepts was not good, so it gave the impression of forcing the completion process based on what he knew, not based on correct data and logical thinking, as a result, S2 did not carry out the evaluation process properly on the results of the completion. So correct mastery of material/concepts will influence the thinking process which will always be correct. Participant S3 was unable to provide an answer to question number 4.

Mathematics and logic are two knowledge that are difficult to separate (Suyitno, 2012), as is the case with integrals which are part of mathematics, in solving problems related to integrals everything is based on the logic of the mind. Participant S1 was able to solve all the problems correctly even though there were some minor errors that did not affect the solution. Based on interviews with S1, it was found that S1 could explain all the researcher's questions related to the concept of integrals to find the area and volume of rotating objects correctly. S1 could explain the indefinite integral of a polynomial function, the integral of a trigonometric function, the integral of a natural logarithm function, the procedure for determining the area under a function curve by applying a definite integral and the procedure for determining the volume of rotating objects using the disc method, the ring method and the cylindrical shell method. So S1's declarative knowledge is very good, as is his procedural knowledge. It is natural that S1 can solve all the problems given by the researcher. This is in accordance with the statement that a problem can be solved effectively if the person must have knowledge of the problem domain (Urban & Urban, 2023). Logical thinking can direct cognition to planning a solution sequentially from what is known, what is asked, what first step should be taken, and what formula should be used. When a problem has been

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successfully identified, metacognition guides the problem-solving process (Aurah et al., 2011). The strategies used by S1 to improve their metacognitive abilities are by (1) reading carefully and trying to understand and comprehend what they read (thinking aloud), (2) doing practice questions to improve metacognitive skills (problem solving activities), (3) making small notes related to concepts so that they can be easier when needed (journal keeping). Learning activities will be easier if there are small notes containing the required material that has not been understood, notes on errors when solving questions and others. This is in accordance with the research results (Cooper & Stevens, 2006) which states that by taking the time to make small notes or journal keeping, learning activities will be more organized, meaningful and professional (4) looking for other learning resources from Google, tutorials on YouTube to explore knowledge, comparing ideas with other ideas (paraphrasing, elaborating and reflecting), (5) planning strategy, (6) identifying learning difficulties (identifying the difficulty), Identifying difficulties in solving integrals is one way to detect learning difficulties early when solving integral problems (Gersten et al., 2005).

Participant S2 could only solve half of the total problems given by the researcher. S2 was only able to solve problems related to integral functions but failed to apply integrals to determine the area and volume of rotating objects. S2's ability to analyze a problem was low, including his mathematical representation ability. This was evident in his failure to represent a problem graphically in question number 2, which was calculating the area of a region using the outer rectangle approach. One important step in problem solving is the ability to represent a problem graphically, which can then be analyzed to find the right solution. To achieve this representation ability requires an understanding and ability that can foster confidence in expressing mathematical thoughts/ideas, this is one of the characteristics of metacognitive abilities (Fadilla & Purwaningrum, 2021). This is in accordance with the statement that metacognitive knowledge can influence metacognitive skills/activities, thereby impacting the resulting achievement (Kusuma & Nisa, 2019). To improve his metacognitive skills, S2 should first improve his declarative, procedural, and conditional knowledge. Based on the results of interviews with S2, it was found that S2's declarative and procedural knowledge were not good, only basic knowledge and basic solution procedures were possessed. For questions that were questions developing basic theories, S2 was unable

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to solve them. His metacognitive skills, which include planning, monitoring, and evaluation, were not carried out well. Planning and monitoring can be carried out well when S2 masters the material asked by the researcher. In other words, metacognitive skills will work well if the person has good declarative, procedural, and conditional knowledge. A person's metacognitive skills can be honed if the person is accustomed to solving a problem, because when someone carries out problem-solving activities, the person will use higher-order thinking skills (Zoller & Pushkin, 2007), including critical and creative thinking. Critical and creative thinking skills must be supported by good metacognitive skills. Thus, metacognitive skills can support higher-order thinking skills, thereby enhancing problem-solving abilities (Güner & Erbay, 2021). This is evident in the fact that S2 is not accustomed to working on practice questions, so when given a problem, S2 cannot determine the strategy to use to solve the problem or uses a strategy that is not appropriate for the given problem. Metacognitive strategies used by S2 to improve their metacognitive abilities include: (1) journal keeping, (2) cooperative learning.

Participant S3 was unable to complete all of the researcher's questions, and there was even one question he failed to answer at all, namely the question about the volume of rotating objects. The errors made by S3 included errors in using known data, such as in question number 1, which were caused by poor solution planning and the absence of a monitoring and re-evaluation process. This occurred due to a lack of confidence in the steps he took. This is in line with Schoenfeld's statement that a person will build their own mathematical framework from their beliefs, intuitions, and past experiences (Pathuddin, 2016). Declarative, procedural, and conditional knowledge are also important for a person to support their metacognitive abilities (Pathuddin et al., 2018). Declarative knowledge plays a role in recalling memories stored in the brain to design a solution plan. S3's declarative, procedural, and conditional knowledge were not good enough, many memories could not be recalled, so S3 seemed to forget and did not understand some of the formulas and rules he had to use. It can be said that even with basic knowledge, S3 still experienced confusion. When asked about how he understood integral material and how he studied it, he said that he rarely sought learning resources other than in class. During classroom learning activities, S3 had difficulty understanding integral material and did not try to find out the things that caused his ignorance. S3 did

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not make an effort to improve his thinking skills, so it can be said that S3 did not do anything to improve his metacognitive abilities.

From the research results, it was found that there is a relationship between metacognitive skills and cognitive strategies, namely the more someone uses metacognitive strategies in learning activities or problem-solving activities, the better their metacognitive skills will be. In accordance with the research results of several experts who stated that there is a relationship between metacognitive skills and cognitive strategies, namely the more someone uses metacognitive strategies in learning activities or problem-solving activities, the better their metacognitive skills (Akamatsu et al., 2019; Wischgoll, 2016; Rivas et al., 2022). The role of implementing metacognitive strategies in learning activities includes: (1) Becoming more prepared and more focused in understanding problems, (2) A never-give-up attitude to find the connection between what is known and what is asked, (3) The ability to dig up information or formulas stored in memory to solve problems, (4) Increasing the ability to correct the right answer, (5) Making someone more accustomed to facing and understanding problems, (6) Becoming accustomed to finding solutions, (7) Making someone more critical and creative in finding problems and solving them, (8) Making someone accustomed to identifying material that requires deeper understanding, (9) Making someone able to identify their learning difficulties, (10) Being able to evaluate knowledge that they do not yet have. This research emphasizes more on tracing cognitive processes and the use of strategies used by students when solving integral problems.

CONCLUSION

Students with strong academic abilities are aware of the necessary steps because they understand the problem well. This is supported by their declarative and procedural knowledge, so their metacognitive abilities function appropriately. These students use several metacognitive strategies. Students with average abilities actually have a good awareness of what they should do, but because their problem understanding and mathematical representation skills are also lacking, they fail to solve the problem correctly at the monitoring stage. These students implement metacognitive strategies, but not intensively. Students with low abilities lack awareness or planning for problem solving, so their monitoring and evaluation skills are not developed. Their problem

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ISSN: 2088-7868, e-ISSN 2502–5708

understanding is also poor, and their declarative and procedural knowledge are also lacking. They do not implement metacognitive strategies.

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ISSN: 2088-7868, e-ISSN 2502–5708