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Transformations Of Students' Mathematical Reasoning In Solving PISA-Like Problems: A Commognitive Perspective

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Abstract

Low performance in international assessments such as the International Student Assessment Program (PISA) highlights the challenges that Indonesian students constantly face when facing non-routine and context-rich mathematical problems. This study aims to explore the transformation of students' mathematical reasoning in solving PISA-like problems using a commonishative perspective. This study uses a descriptive qualitative approach. The subject of this study involves 2 students who were selected based on the results of interviews with the teaching teacher. The subject selection technique used is purposive sampling. Data collection in this study uses tests, interviews and observations. The validity of the data used is triangulation method, namely by looking at test results, interviews, and observations. This research was conducted in one of the high schools in Central Java. The data analysis used in this study includes data collection, data reduction, data presentation, and conclusion drawing. The findings of the study showed that the first student successfully met all the indicators of mathematical reasoning, including making written mathematical statements, predicting answers, compiling evidence through rebuttal arguments, generalizing spatial relationship patterns, and drawing valid conclusions after going through the cognitive restructuring process. On the other hand, the second student only met the indicators of making initial mathematical statements, predicting answers, and refuting concepts through verbal mediation, but failed to meet the indicators of pattern generalization, preparation of proof steps, and drawing conclusions due to dependence on mechanical procedures and confusion of mathematical dimensions.

Keywords: Mathematical Reasoning, PISA-like Problems.

Transformasi Penalaran Matematis Siswa Dalam Menyelesaikan Masalah Mirip PISA: Perspektif Komognitif

Abstrak

Kinerja rendah pada penilaian internasional seperti Programme for International Student Assessment (PISA) menyoroti tantangan yang terus-menerus dihadapi siswa Indonesia saat menghadapi masalah matematika yang tidak rutin dan kaya konteks. Penelitian ini bertujuan untuk mengeksplorasi transformasi penalaran matematis siswa dalam menyelesaikan soal mirip PISA dengan menggunakan perspektif

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komognitif. Studi ini menggunakan pendekatan kualitatif deskriptif. Subjek penelitian ini melibatkan 2 siswa yang yang dipilih berdasarkan hasil wawancara dengan guru pengampu. Teknik pemilihan subjek yang digunakan adalah purposive sampling. Pengumpulan data pada penelitian ini menggunakan tes, wawancara dan observasi. Keabsahan data yang digunakan adalah triangulasi metode yakni dengan melihat hasil tes, wawancara, dan observasi. Penelitian ini dilakukan di salah satu SMA di Jawa Tengah. Analisis data yang digunakan pada penelitian ini meliputi pengumpulan data, reduksi data, penyajian data, dan penarikan kesimpulan. Temuan penelitian menunjukkan bahwa siswa pertama berhasil memenuhi seluruh indikator penalaran matematis, meliputi pembuatan pernyataan matematis tertulis, prediksi jawaban, penyusunan bukti melalui argumen sanggahan, generalisasi pola hubungan spasial, hingga penarikan kesimpulan yang valid setelah melalui proses restrukturisasi kognitif. Sebaliknya, siswa kedua hanya memenuhi indikator pembuatan pernyataan matematis awal, prediksi jawaban, dan sanggahan konsep melalui mediasi verbal, namun gagal memenuhi indikator generalisasi pola, penyusunan langkah pembuktian, serta penarikan kesimpulan akibat ketergantungan pada prosedur mekanis dan kerancuan dimensi matematis.

Kata kunci: Penalaran matematika, Soal mirip PISA.

INTRODUCTION

The results of the Programme for International Student Assessment (PISA) study show that the achievement of mathematical literacy of Indonesian students is still relatively low (Wijaya et al., 2024). A number of studies reveal that many Indonesian students have difficulty working on questions in the PISA test (Bayirli et al., 2023; Fenanlampir et al., 2019; Stacey, 2011; Thien et al., 2015) . The PISA questions themselves are designed not only to measure the understanding of mathematical concepts, but also to assess the extent to which students are able to apply the concepts in situations that are practical, dynamic, and in accordance with the real-life context (Stacey & Turner, 2015). To be able to solve this kind of problem, students are required not only to rely on standard procedures, but also need to develop the reasoning that students have (Yansen et al., 2019).

In this 21st century era, the ability to reason mathematically is one of the main skills that students need to have in order to solve various mathematical problems, especially in facing the complexity of real-world challenges (Palinussa et al., 2021). A

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solid foundation of mathematical reasoning is essential in understanding various mathematical concepts while solving problems that arise in various real-life contexts (Sun et al., 2025) . Furthermore, mathematical reasoning provides the foundation for logical thinking, problem-solving, and decision-making (Herbert & Williams, 2023) . Students' mathematical reasoning skills can be honed through habituation in working on mathematical problems that require a reasoning process, one of which is PISA-type problems (Kamaliyah et al., 2013) . However, the use of PISA-type math problems in schools to hone students' mathematical reasoning skills is still not carried out optimally (Noor Kholid et al., 2022). This condition certainly needs improvement (Setianingsih et al., 2024) . So far, mathematics learning tends to focus on mastering concepts and applying formulas to solve problems that are less related to real-life contexts (Wijaya et al., 2024; Zulkardi & Kohar, 2018).

Based on the results of the initial observations carried out, it is shown that students experience serious obstacles when faced with non-routine contextual mathematics tasks equivalent to PISA. In the observation of real-context geometry tasks, students tend to rely on visual routines such as simply counting points without understanding spatial concepts and underlying mathematical relationships. students who had drawn both curved and straight solutions failed to notice the clear difference in their lengths until their attention was explicitly directed to it (Haataja et al., 2025) . When presented with problems that require graded modeling and reasoning, students often experience dimensional confusion and difficulty in transmitting visual representations into valid formal mathematical arguments.

To bridge these problems, this study designed an in-depth qualitative investigation by utilizing PISA-like problems. Solving problems similar to PISA questions can boost students' confidence when facing actual PISA questions (Nurazizah & Zulkardi, 2022). The basis of thinking behind this analysis rests on the understanding that the low performance of students' mathematical reasoning is not an innate cognitive deficit that is static, but a reflection of the communicatively immature mathematical thinking discourse. Therefore, the Sfard comcognitive framework is applied in detail to dissect the evolution of students' reasoning through its four main components, namely Word Uses, Visual Mediators, Routines, and Narratives. Through this discursive mapping, conceptual barriers and the process of students' cognitive restructuring can be identified

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precisely, thus providing a strong empirical foundation in transforming mathematics learning in schools from mechanical algorithmic exercises to a dialogical learning environment based on real problem-solving.

Research conducted by (Hamidah et al., 2025) with a mixed method approach found that although prospective mathematics teachers are relatively strong in formulating conjectures, they still face persistent challenges in generalizing and justifying. Another study was conducted by (Turgut, 2022) which explained that students' initial exploration began with dynamic shifting of the slider, which led them to visual observations of the concepts of identity transformation and dilatation. Most of the research that has been conducted has focused on analyzing the results of mathematical reasoning. This research comes with a different approach because it not only describes the results but also analyzes the process of mathematical reasoning transformation using a commognitive framework.

To explore changes in students' mathematical reasoning when solving PISA-like problems, this study applied (Sfard, 2008) commognitive framework, which views mathematical reasoning as an observable discursive activity in which language plays a central role. From this perspective, mathematical thinking emerges through interactions between individuals and various mediators, including words, visual representations, narratives, and routines (Cooper & Lavie, 2021) . The commognitive framework can facilitate students' cognitive processes when solving PISA-type questions (Yansen et al., 2019) . In cognitive theory, thinking is defined as an individualized and self-directed discourse. Mathematical reasoning is an important ability to have, especially in mathematics learning. Previous studies on mathematical reasoning have focused too much on the achievement of indicators (Gustiadi et al., 2021; Lestari et al., 2022; Luti et al., 2022; Muniroh & Buchori, 2024; Nurazizah & Zulkardi, 2022; Salam et al., 2023; Sumardi & Amalia, 2021) . This research not only focuses on indicators but also analyzes how students transform using the commognitive framework. The purpose of this study is to describe the transformation mathematical reasoning observed in 15-year-old Indonesian students when solving PISA-type questions, using the commognitive framework perspective.

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RESEARCH METHODS

The approach in this study is qualitative with a descriptive design. The qualitative approach is designed to explore in depth the mathematical reasoning process of students when presented with a PISA-like problem. The data collection technique in this study involves tests, observations, and interviews with research subjects. The data were described and analyzed in depth using mathematical reasoning indicators from (NCTM, 2000), which were then further developed by the researchers into several sub-indicators. The material chosen to be the basis for making test questions in this study is geometry. The data analysis used in this study follows the guidelines from (Miles & Huberman, 1994) which include data collection, data reduction, data presentation, and conclusion drawn.

The first indicator of mathematical reasoning is to make a written mathematical statement consisting of 2 sub-indicators developed: (1) Stating the relationship between mathematical elements, (2) converting the information of the problem into the form of a mathematical statement. The second indicator is to predict the answer consisting of 2 sub-indicators developed: (1) Predicting the tendency or providing an initial response to the truth of a claim/statement, (2) Formulating a hypothesis/estimate regarding the adequacy of data or the measurability of an object. The third indicator, namely Compiling Proof, consists of 2 sub-indicators developed: (1) Writing down the steps for completion, (2) Showing rebuttal arguments to prove the fallacy of a concept. The fourth indicator, namely Using Patterns and Relationships to Compile Estimates, consists of 2 sub-indicators developed: (1) Identifying the order of the arrangement, (2) Generalizing the pattern of the arrangement of participants. The fifth indicator is to draw conclusions consisting of 2 sub-indicators developed: (1) Determining the validity/validity of a statement or calculation result based on mathematical rules., (2) Formulating a logical final conclusion sentence that answers the problem of the real context. The development of sub-indicators based on indicators makes it easier for researchers to analyze students' mathematical reasoning processes. In addition, the analysis in this study was conducted with commognitive components which include Word Uses (WU), Visual Mediators (VM), routines (R), and narratives (N).

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The subjects in this study are grade XI students at one of the high schools in Central Java. The selection of this subject was based on the results of interviews with the teacher who taught it, where the teacher nominated that this subject had good mathematical reasoning. This is in line with the targeted sampling strategy commonly used in theory development case studies, where participants are selected for their potential in providing in-depth and information-rich data (Yin, 2018). The selection of subjects in this study uses purposive sampling which is selected according to the criteria that have been determined. The subjects in this study were two people who were determined based on the criteria that had been described earlier.

This stage of research begins with the identification of problems that occur in students' mathematical reasoning. After the problem is identified, it is continued with a theoretical study to determine the right indicators and forms of analysis for this research. The next stage is continued with the creation of a research instrument where the validation chosen is content validation involving experts to assess the validity of this research instrument. After the instrument is declared valid, the next step is the implementation of the research. The implementation of the research is carried out with the aim of collecting data.

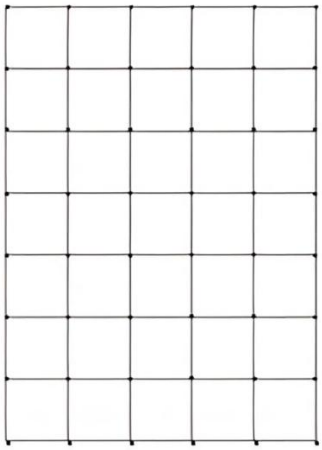
The data collection techniques in this study include tests, interviews, and observations. The instruments used were PISA-like test questions, interview guidelines, and observation sheets. The questions developed in this study have been adjusted to real problems that occur in the real world. The validation of the questions in this study was carried out through a direct expert review that provided a qualitative assessment of its suitability with the characteristics of PISA, especially the aspects of realism and the necessary cognitive processes (OECD, 2023). Interviews are conducted in a semi-structured manner which means follow-up questions can develop according to the student's answers. This is done to dig deeper into the existing information. Furthermore, observation is carried out with the aim of observing every movement, expression, and action of the participants. All instrument validation was carried out by involving experts, namely 1 professor and 1 associate professor in the field of mathematics education. To ensure the wetness of the data in this study, we used the triangulation method. The questions resembling PISA can be seen in Table 1.

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Table 1. PISA-Like Mathematical Problems

PISA-Like Mathematical Problems
At the Festival Jaga Jarak: Ajang Berkreasi Para Seniman Belia (Social Distancing Festival: A Creative Platform for Young Artists) held in Parachiyangan Field, Majalengka Regency, West Java, the field allocated for festival participants is rectangular in shape, as illustrated below.





To reduce the possibility of violating the rules, each participant is required to implement social distancing during the activity. The committee has set a number of points on the field as sitting positions for participants, with the aim that the distance between participants is maintained as far as one meter. Based on the position arrangement, the field is estimated to be able to accommodate as many as 48 participants. To answer the following questions, follow the steps in the activity below:

a) In the context of sponsorship cooperation, the committee plans to install billboards around the area where participants sit, except in the front which is designated as a reception area and the judges' place. Before installing the board, the committee needs to know in advance the surroundings of the participant's area. One of the committee members tried to calculate the circumference of the area by adding up all the points that had been marked around it, and the result was 24 meters. Do you think the calculation is correct? If you don't think it's right, describe the method that should be used to get the correct circumference, and then determine the result.

b) Calculate the total length of the billboard that must be prepared by the committee. It is known that the distance between the participant's area and the billboard is 1 meter on each side, except on the front side which must be 2 meters apart.

c) The committee also needs data on the size of the festival field to be included in the final report of the activity. Based on the information available in the question, can the area of the festival field be calculated? If possible, determine the area, then look for a comparison between the area of the participants and the overall area of the festival field. Describe each step of your calculation in detail.

d) If the committee is also asked to determine the circumference of the entire festival field, describe the steps that can be taken to obtain it. Provide a complete and detailed explanation of the process.

After the questions are given, the data collected is analyzed using indicators

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from (NCTM, 2000) which have been developed and further studied through a commognitive perspective (Sfard, 2008). Details of the indicators used can be seen in Table 2.

Table 2. Indicators of mathematical reasoning

Mathematical Reasoning Indicators	Sub Indicators of mathematical reasoning	Code	Commognitive
Create a math statement in writing	Stating the relationship between mathematical elements.	A1	(WU), (VM), (R), (N).
	Convert the information of the problem into the form of a mathematical statement.	A2	(WU), (VM), (R), (N).
Predicting the answer	Predicting trends or giving an initial response to the truth of a claim/statement.	B1	(WU), (VM), (R), (N).
	Formulate hypotheses/estimates regarding the adequacy of data or the measurability of an object.	B2	(WU), (VM), (R), (N).
Compiling Evidence	Write down the solution steps	C1	(WU), (VM), (R), (N).
	Showing a rebuttal argument to prove the fallacy of a concept.	C2	(WU), (VM), (R), (N).
Using Patterns and Relationships to Construct Guesses	Identify the order of the arrangement.	D1	(WU), (VM), (R), (N).
	Generalization of the arrangement pattern.	D2	(WU), (VM), (R), (N).
Drawing conclusions	Determine the validity/validity of a statement or calculation result based on mathematical rules.	E1	(WU), (VM), (R), (N).
	Formulate a logical final conclusion sentence that answers the problem of real context.	E2	(WU), (VM), (R), (N).

RESULTS AND DISCUSSION

The selection of subjects in this study was based on the assessment of the teaching teacher who stated that these 2 subjects had better cognitive abilities than others in one class. The results of data analysis obtained through PISA-like problem-solving tests, semi-structured interviews, and observations of S3 and S4 subjects were elaborated in depth based on mathematical reasoning indicators integrated with four components of commognive discourse: Word Uses (WU), Visual Mediators (VM), Routines (R), and Narratives (N).

Transformations in S3's Reasoning for Task (a)

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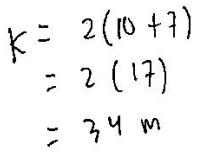
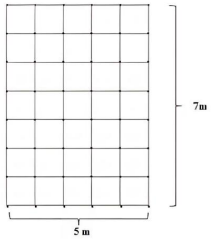
Original version	Translated version
	$K = 2(10 + 7)$ $= 2\ (17)$ $= 34$

Figure 1. Task Answer Sheet (a)

Based on Figure 1, it can be seen that S3 evaluated the organizer's claim regarding a perimeter of 24 meters. Initially, S3 determined the dimensions by counting grid units using the diagram provided in the problem. Codes A1 (stating relationships between mathematical elements) and A2 (converting information to mathematical statements) were initially attempted. The subject expressed agreement based on a routine calculation of grid spaces 2 × (7+5) = 24 m. However, upon re-evaluating the problem's spatial representation and prompt rules, S3 initiated a cognitive shift. S3 realized that counting grid boxes differed from counting boundary points versus intervals. Codes B1 (predicting trends/evaluating claims) and C2 (providing counter-arguments to refute incorrect concepts) were demonstrated effectively. The interview transcript reflects this cognitive restructuring:

R : Could you explain your answer to question (a)?
S3 : It asks to determine the perimeter... first, I found the length and width by counting grid intervals, which were 7 and 5, then using $2 \times (p + l) = 24$, I got 24.
R : What concept did the committee use?
S3 : One member counted points directly. But points are positions, while perimeter requires summing the actual distances between outer points.

S3 revised the response by properly converting the problem context into an appropriate mathematical form (A2, C1). The Word Uses (WU) applied included "length," "width," "interval," and "perimeter". The Visual Mediators (VM) shifted from simple grid counting to identifying point-to-point distance segments.



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Figure 2. Subject's calculation process and visual representation for task (a)

Based on Figure 2, this study found that the significant transformation in the mathematical reasoning structure of students when dealing with PISA-type contextual problems. Instead of reflecting static or fragmented cognitive abilities, subjects (S3) exhibited adaptive and dynamic discourse trajectories. In the early stages, the problem-solving process was dominated by visual-empirical reasoning and intuitive informal routines. However, as the complexity of the intervention and the demands of the task increased, the discourse transitioned towards a formal articulation characterized by the use of appropriate symbolic representations, generalization of rules, and validation of mathematically coherent arguments.

At the initial stage of solving task (a), S3 evaluated the organizer's claim regarding a perimeter of 24 meters by stating relationships between elements (A1) and converting problem information (A2). S3 relied on ritualized routines, such as counting discrete grid points or superficial spatial boundaries, which led to incorrect initial predictions regarding perimeter and data sufficiency. However, when confronted with contextual constraints and guided by dialogic probing during interviews, S3 underwent a cognitive restructuring. This transformation was evidenced by shifts across all four commognitive components: Word Uses (WU) transitioned from informal spatial descriptors like "grid points" to precise mathematical terminology such as "interval" and "perimeter"; Visual Mediators (VM) evolved from basic grid counting to structured geometric schemas; Routines (R) shifted from node-counting to formal algebraic calculations ($K = 2(p + l)$) and Narratives (N) matured into substantiated, contextually valid conclusions.

The Routines (R) involved transition from counting discrete nodes to applying the formal perimeter routine $K = 2(p + l)$. The Narratives (N) constructed by S3 were rooted in the formal mathematical definition of perimeter in real-world contexts (E1, E2).

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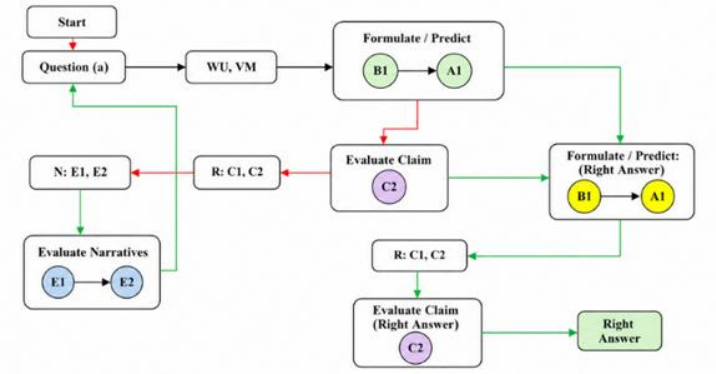


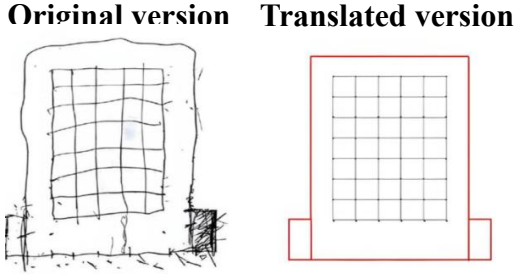
Figure 3. Flow of changes in S3's mathematical reasoning pattern for task (a)

Based on Figure 3, it can be concluded that the mathematical reasoning pattern of S3 in task (a) has changed from an initial wrong answer to a correct answer. The flow starts from Start, where the question is given to the student, then S3 reads Question (a) and understands the problem and makes representations (WU, VM). In the Formulate/Predict stage, S3 shows the codes B1 and A1, which are making predictions and stating the relationships between mathematical elements based on the routine calculation of grid space ($2 \times 7 + 5 = 24$ m).

Table 3. The remarks of codes in Figure 3

Graphic Codes	Remarks	Graphic Codes	Remarks
Start	Questions are given to students.	E1	Determine the validity/validity of a statement or calculation result based on mathematical rules.
Questions (a)	Problems that students will work on in section (a)	E2	Formulate a logical final conclusion sentence that answers the problem of real context
A1	Stating the relationship between mathematical elements	WU	Word Uses
B1	Predicting trends or giving an initial response to the truth of a claim/statement	VM	Visual Mediators
C1	Write down the solution steps	N	Narratives
C2	Showing a rebuttal argument to prove the fallacy of a concept.	R	Routines

Transformations in S3's Reasoning for Task (b)



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Figure 4. Subject's initial sketch for task (b)

Based on Figure 4, Subject S3 can identified the information given regarding the advertising board placement: a 1-meter distance on the left, right, and back sides, and a 2-meter distance on the front side (where no boards are placed). In the first attempt, S3 attempted to draw a boundary sketch around the area. This initial spatial representation led S3 to perform a superficial line-by-line counting routine, resulting in an incorrect estimation of 37 meters. This obstacle reflects how students frequently rely on familiar procedural or arithmetic-based counting routines when navigating non-routine problem contexts, rather than immediately deploying abstract spatial comparison.

S3 initially calculated the perimeter as 37 meters by summing arbitrary outer segments (E1, E2 executed, but C1, D1 misapplied). S3 struggled to articulate the reasoning during the interview until prompted to reflect on the front boundary rule:

R : How did you calculate the total length of the board?
S3 : I calculated each outer segment one by one, getting 37 meters.
R : Does this calculation also include the front side reserved for reception and judges?
S3 : Oh... no! The front side must remain open. It should only be the length of the boards covering the three remaining sides!

When prompted during the interview to reflect on the contextual rule that the front area must remain open for reception and judges, S3 underwent a clear discursive shift. The student reorganized the visual mediators (VM) and constructed a revised 3-sided open rectangle schema with expanded boundaries. This transition illustrates the vital role of dialogic probing and formative teacher scaffolding in eliciting higher-level mathematical reasoning and guiding students toward self-correction (Smit et al., 2019). This dialogic prompt triggered a shift in S3's Visual Mediators (VM) and Routines (R). Subject S3 re-examined the spatial layout and constructed a revised visual schema (D1, D2).

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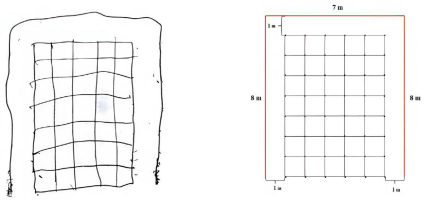


Figure 5. Revised answer schema of the subject for task (b)

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Based on Figure 5, Subject S3 successfully wrote the step-by-step proof (C1), generalized the spatial pattern (D2), and drew a logical contextual conclusion (E2). The Word Uses (WU) were refined to distinguish "participant perimeter" from "open advertising perimeter". The cognitive flow transition is mapped in Figure 6.

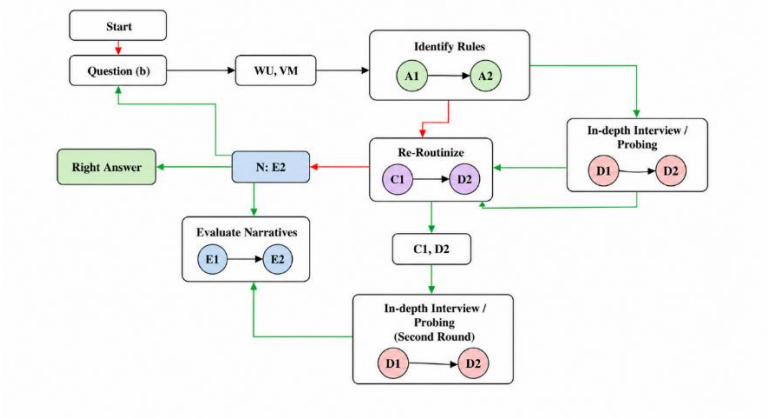


Figure 6. Flow of S3's changing mathematical reasoning process for task (b)

Based on Figure 6, it can be concluded that the S3 mathematical reasoning pattern in task (b) has changed from the application of incorrect routine procedures to correct answers through the help of in-depth interviews. The flow starts from Start, which is a question given to students, then S3 reads Question (b) and understands the problem and makes a representation (WU, VM). In the Identify Rules stage, S3 shows the A1 and A2 codes, which identify the information and rules given in the question, such as the placement distance of the billboard.

Table 4. The remarks of codes in Figure 6

Graphic Codes	Remarks	Graphic Codes	Remarks
Start	Questions are given to students.	E1	Determine the validity/validity of a statement or calculation result based on mathematical rules.
Questions (b)	Problems that students will work on in section (a)	E2	Formulate a logical final conclusion sentence that answers the problem of real context
A1	Stating the relationship between mathematical elements	WU	Word Uses
A2	Convert the information of the problem into the form of a mathematical statement.	VM	Visual Mediators
C1	Write down the solution steps	N	Narratives
D1	Identify the order of the arrangement	R	Routines
D2	Generalization of the arrangement pattern		

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Transformations in S3's Reasoning for Task (c)

In task (c), S3 initially claimed that the total area of the festival field could not be calculated due to insufficient data regarding total field boundaries (B2). However, after revising task (b), S3 realized that the bounded festival field area could be defined using the extended board dimensions.

R : In question (c), you wrote that the total area could not be determined. Can you explain?

S3 : After revising question (b), I realized the festival area bounded by the setup has a length of 10 m and width of 7 m.

R : How did you compute the area ratio?

S3 : Participant area $7 \times 5 = 35m^2$. Festival area $10 \times 7 = 70m^2$.. So the ratio is 35 :70, which simplifies to 1 : 2.

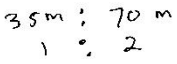
Original version	Translated version
	35: 7 1:2

Figure 7. Revised response sheet of S3 for task (c)

Based on Figure 7, Subject S3 executed sub-indicators B2 (hypothesizing data sufficiency), C1 (writing solution steps), and E2 (drawing valid ratio conclusions). Mathematical reasoning in problem-solving environments relies heavily on the interplay between inscriptions and discursive mediation, where shortcomings in initial representations are complemented by reflective reasoning.

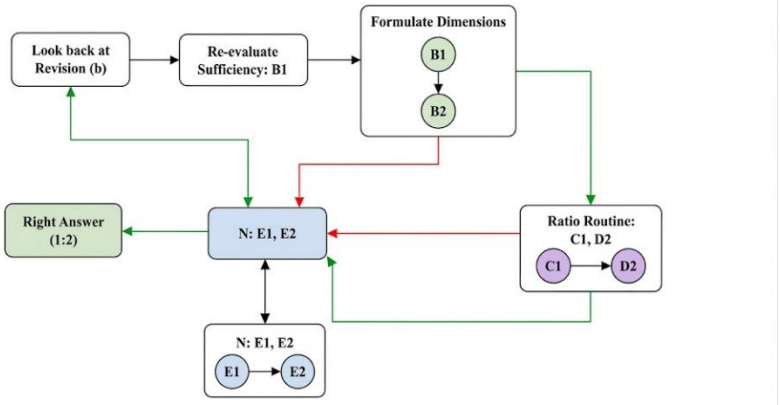


Figure 8. Flow of subject's cognitive transformation for task (c)

Based on Figure 8, it can be concluded that the mathematical reasoning pattern of S3 in task (c) undergoes a cognitive transformation from an erroneous initial claim to a

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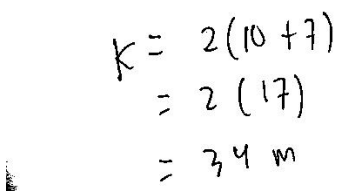
correct answer (1:2) through a process of reviewing the results of the work in task (b). In the initial stage, S3 states that the total area of the festival area cannot be calculated because the data of the overall field boundary is considered insufficient (B2). This claim is then evaluated through *the Ratio Routine* (C1, D2), i.e. S3 uses the ratio procedure routinely and generalizes the arrangement pattern, which is then checked for validity through narrative (N: E1, E2).

Table 5. The remarks of codes in Figure 8

Graphic Codes	Remarks	Graphic Codes	Remarks
Look back at Revision (b)	Students double-check on question (b)	D2	Generalization of the arrangement pattern
Re-evaluate Sufficiency	Students re-evaluate	E1	Determine the validity/validity of a statement or calculation result based on mathematical rules.
B1	Predicting trends or giving an initial response to the truth of a claim/statement	E2	Formulate a logical final conclusion sentence that answers the problem of real context
B2	Formulate hypotheses/estimates regarding the adequacy of data or the measurability of an object	N	Narratives
C1	Write down the solution steps	R	Routine

Transformations in S3's Reasoning for Task (d)

Original version



Translated version

K = 2 (10+7)
= 2 (17)
= 34 m

Figure 9. Revised answer sheet of S3 for task (d)

Based on Figure 9, Subject S3 relied on the conceptual dimensions established during task (c) $(p = 10, l = 7)$ to determine the perimeter of the entire festival field. As (Hughes et al., 2020) emphasize, an individual's ability to communicate mathematical reasoning effectively in written form relies on the precise integration of algebraic representations, structured solution steps, and valid mathematical outcomes.

S3 executed sub-indicators C1 (writing execution steps) and E1 (determining valid mathematical outcomes) with high fluency. The sequence of cognitive progression for task (d) is summarized in Figure 10.

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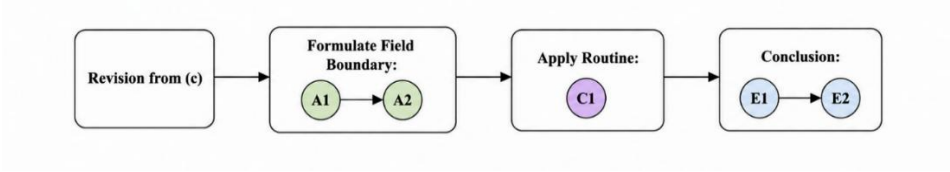


Figure 10. Flow of cognitive transformation for task (d)

Based on Figure 10, it can be seen that the mathematical reasoning pattern of S3 on task (d) runs smoothly and linearly, without significant cognitive obstacles. The flow begins with Revision from (c), where S3 utilizes the results of the correction of the answers in task (c) as the basis for completing task (d). This shows that the understanding that has been improved previously is carried over and used effectively in the next problem.

Table 6. The remarks of codes in Figure 10

Graphic Codes	Remarks	Graphic Codes	Remarks
Revision	Students make corrections to answers	E1	Determine the validity/validity of a statement or calculation result based on mathematical rules.
A1	Stating the relationship between mathematical elements	E2	Formulate a logical final conclusion sentence that answers the problem of real context
A2	Convert the information of the problem into the form of a mathematical statement.	R	Routine
C1	Write down the solution steps		

Transformations in S4's Reasoning for Task (a)

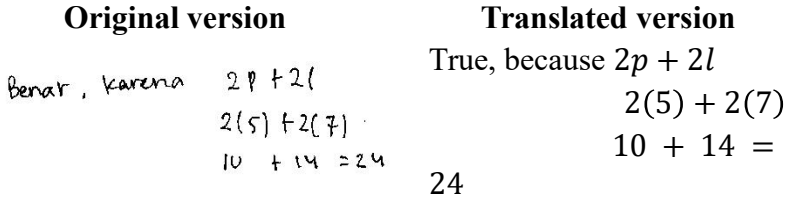


Figure 11. Task Answer Sheet (a)

Based on Figure 11, Subjek S4 was asked to critique the event organizer's claim that the perimeter of the designated seating zone was 24 meters. Initially, S4 attempted to establish mathematical statements (sub-indicators A1 and A2) by relying on the visual layout of the grid. The student accepted the organizer's 24 m figure through a superficial routine of adding perimeter segments derived directly from counting grid squares 2(5)+2(7)=24.

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S4 executes Routine (R) calculation of the circumference of a formal rectangle $K = 2p + 2l$ correctly. The Word Uses (WU) used are relatively formal, involving the variables p, l, and standard algebraic operations. Visual Mediators (VMs) that S4 utilizes are limited to extracting dimensions from visual grids without evaluating the difference between discrete nodes and boundary segment lengths (intervals). As a result, in the indicators of evaluating claims (B1) and drawing conclusions (E1, E2), S4 produces a narrative (N) that is procedurally algebraically correct $(2(5) + 2(7) = 24)$. However, conceptually-spatially, it is less critical because it accepts the committee's initial claim without any proof of rebuttal. To strengthen the results of the analysis of the answers from the S4 subjects, we present the following interview results:

R : Can you clarify how you arrived at your conclusion for Task (a)?
S4 : At first, I measured the dimensions by counting grid intervals, obtaining 7 meters and 5 meters. Substituting these into $2p + 2l$, I got 24 meters.
R : What was the underlying assumption of the committee's calculation?
S4 : They directly counted the marked points on the perimeter. However, points represent position markers, whereas perimeter represents the continuous distance along the boundary.

These interview findings confirm the cognitive restructuring experienced by the participant when re-evaluating the perimeter context. Through verbal mediation, S4 explicitly refuted the committee's point-counting methodology by highlighting the conceptual flaw of treating spatial points as linear distance measurements. This shift demonstrates how dialogic scaffolding triggers the evolution of Word Uses (WU) and Narratives (N), ultimately transforming an uncritical acceptance into a mathematically sound, contextual justification. The overall flow of cognitive transformation for task (a) is illustrated in Figure 12.

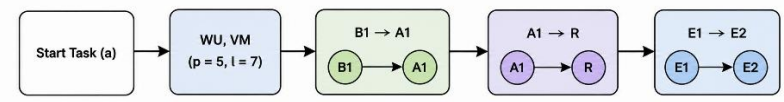


Figure 12. Flow of changes in S4's mathematical reasoning pattern for task (a)

Based on Figure 12, it can be concluded that the mathematical reasoning pattern of S4 in task (a) changes from an uncritical acceptance of the claim to a mathematically correct contextual justification through the help of verbal mediation. The flow begins with Start Task (a), where S4 starts working on the problem. Next, S4 gives rise to

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Word Uses and Visual Mediators (WU, VM) with values p = 5 and l = 7, which shows that S4 uses words and visual representations to understand the field size information in the problem.

Table 7. The remarks of codes in Figure 11

Graphic Codes	Remarks	Graphic Codes	Remarks
Start Task	Students start working on the	E2	Formulate a logical final conclusion sentence that answers the problem of real context
A1	Stating the relationship between mathematical elements	WU	Word Uses
B1	Predicting trends or giving an initial response to the truth of a claim/statement	VM	Visual Mediators
E1	Determine the validity/validity of a statement or calculation result based on mathematical rules.	R	Routine

Transformations in S4's Reasoning for Task (b)

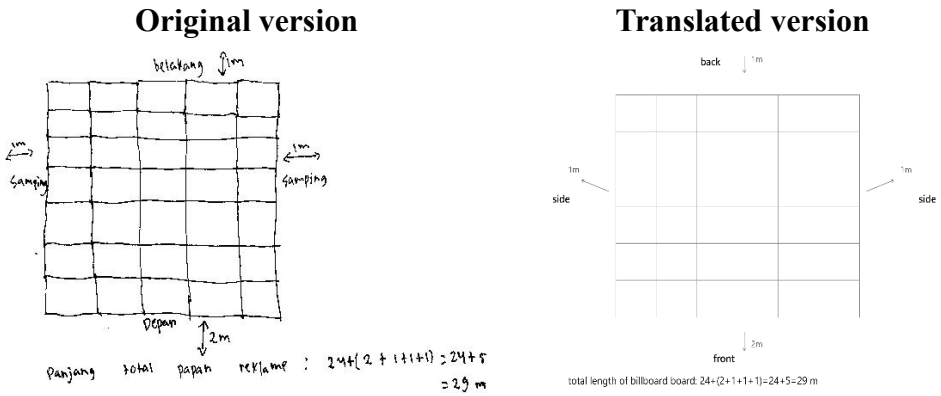


Figure 13. Subject's initial sketch for task (b)

Based on Figure 13, S4 succeeded in identifying the offset rules listed in the question, namely a distance of 1 meter for the back and sides, and 2 meters for the front side (A1, A2). This is shown through the preparation of Visual Mediators (VM) in the form of a re-image representation of the seating area grid equipped with direction labels and distance marker arrows (rear 1m, side 1m, front 2m). Although the initial visual scheme was successfully constructed, S4 experienced cognitive obstacles in generalizing the pattern of spatial boundary expansion (D1, D2). Instead of restructuring

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the new length and width dimensions of the outer boundary and emptying the front side, the S4 instead applied a simple scalar additive Routine (R). S4 summed the circumference of the initial sitting area (24 m) from Task (a) by the accumulation of the number of additional distances (2 + 1 + 1 + 1 = 5), thus obtaining a final result of 24 + 5 = 29 m. This behavior suggests that the shift in Word Uses (WU) on spatial labels has not been followed by a complete geometric reasoning transformation, so that the resulting Narrative (N) and conclusion (E2) 29 m are not spatially valid. The overall flow of cognitive transformation for task (b) is illustrated in Figure 14.

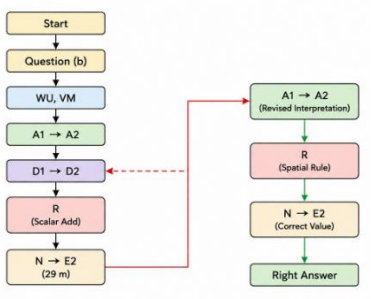


Figure 14. Flow of changes in S4's mathematical reasoning pattern for task (b)

Based on Figure 14, it can be concluded that the mathematical reasoning pattern of S4 in task (b) changed from an incorrect scalar addition (29 m) to the correct answer after S4 revised his interpretation of spatial rules. The flow starts from Start, where the question is given to students, then S4 reads Question (b) about determining the total length of the billboard and understanding the problem and making representations (WU, VM). In the next stage, S4 performs A1 → A2, which is to state the relationship between mathematical elements (A1) and convert the information in the problem into a mathematical statement (A2).

Table 8. The remarks of codes in Figure 13

Graphic Codes	Remarks	Graphic Codes	Remarks
Start	Students start working on the problem	D2	Generalization of the arrangement pattern
Question (b)	Question section (b)	WU	Word Uses
A1	Stating the relationship between mathematical elements	VM	Visual Mediators
A2	Convert the information of the problem into the form of a mathematical statement.	N	Narratives
D1	Identify the order of the arrangement	R	Routine

Transformations in S4's Reasoning for Task (c)

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In Task (c) which asked for an analysis of the adequacy of field area data and the determination of area ratio, S4 predicted that the field area could be calculated ("Can") by referring to the capacity information of 48 participants (B2). However, in the preparation of the proof steps (C1, C2), S4 experienced a shift in the meaning of words (word use) and conceptual confusion between the magnitude of the dimensions.

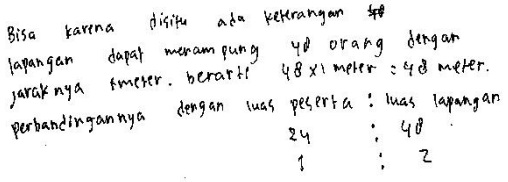
Original version	Translated version
	could be because there is information that 48 fields can accommodate 48 people with a distance of 1 meter. Means 48×1 meters = 48 meters. Comparison with 24 : 48 1 : 2

Figure 15. Task Answer Sheet (c)

Based on Figure 15, Subject S4 exhibited a clear conceptual drift and dimensional confusion during the problem-solving process. S4 equated the physical capacity of 48 participants directly to linear distance by performing $48 \times 1\ meter$ = 48 meter, which was subsequently mislabeled as the "field area". The overall flow of cognitive transformation for task (c) is illustrated in Figure 16.

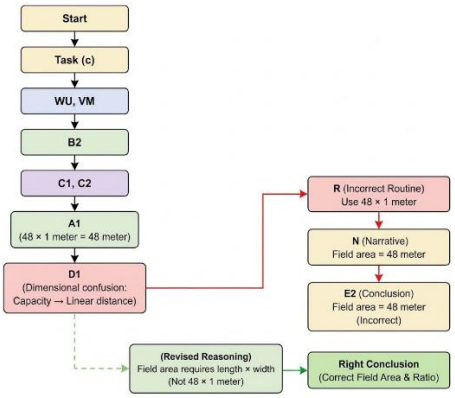


Figure 16. Flow of changes in S4's mathematical reasoning pattern for task (c)

Table 9. The remarks of codes in Figure 16

Graphic Codes	Remarks	Graphic Codes	Remarks
Start	Students start working on the problem	E2	Formulate a logical final conclusion sentence that answers the problem of real context
Task (c)	Question section (c)	WU	Word Uses
A1	Stating the relationship between mathematical elements	VM	Visual Mediators

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B2	Formulate hypotheses/estimates regarding the adequacy of data or the measurability of an object	N	Narratives
C1	Write down the solution steps	R	Routine
C2	Showing a rebuttal argument to prove the fallacy of a concept.	Right Conclusion	Students make conclusions correctly
D1	Identify the order of the arrangement	Revised Reasoning	Students improve their reasoning skills

Transformations in S3's Reasoning for Task (d)

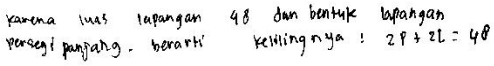
Original version	Translated version
	Because of the area of the field 48 and the shape of the square field. Means the circumference: $2p + 2l = 48$

Figure 17. Task Answer Sheet (d)

Based on Figure 17, S4 utilizes the results of the dimensional assumptions from Task (c) (A1, A2). Subject S4 states that since the area of the field is 48 and rectangular, its circumference can be expressed in mathematical equations.

This step reveals a severe dimensional misconception and a breakdown in S4's Word Uses (WU) and Routines (R). S4 conflated the scalar concept of area $48m^2$ directly with the linear boundary formula for perimeter $(2p + 2l)$, substituting a spatial area quantity into the position designated for total perimeter length. Furthermore, S4's Routine (R) remained incomplete (C1); the participant halted after setting up the algebraic equation without attempting to isolate variables, determine explicit values for length p and width l, or compute a final numerical perimeter. As a result, the constructed Narrative (N) (E1, E2) represents an unvalidated, incomplete mathematical claim (incomplete narrative). This finding demonstrates that while S4 could mechanically recall formal algebraic structures $(2p + 2l)$, the lack of integrated Visual Mediators (VM) and conceptual clarity prevented the student from executing a valid multi-step problem-solving trajectory in context-rich PISA-like tasks. The overall flow of cognitive transformation for task (d) is illustrated in Figure 18.

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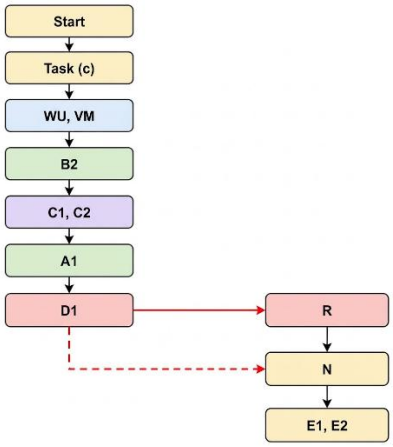


Figure 17. Flow of changes in S4's mathematical reasoning pattern for task (d)

Based on Figure 17, it can be concluded that the mathematical reasoning pattern of S4 in task (d) did not succeed in achieving a valid solution, because the reasoning flow stopped at the reflection and narrative stage without going through adequate evaluation. The flow started from Start, then S4 used the results of Task (c) as a basis for working on task (d) about determining the circumference of the entire festival field. Next, S4 gave rise to Word Uses and Visual Mediators (WU, VM) to understand the information in the problem.

Table 9. The remarks of codes in Figure 18

Graphic Codes	Remarks	Graphic Codes	Remarks
Start	Students start working on the problem	E1	Determine the validity/validity of a statement or calculation result based on mathematical rules.
Task (c)	Question section (c)	E2	Formulate a logical final conclusion sentence that answers the problem of real context
A1	Stating the relationship between mathematical elements	WU	Word Uses
B2	Formulate hypotheses/estimates regarding the adequacy of data or the measurability of an object	VM	Visual Mediators
C1	Write down the solution steps	N	Narratives
C2	Showing a rebuttal argument to prove the fallacy of a concept.	R	Routine
D1	Identify the order of the arrangement		

The findings of this study underline that students' mathematical reasoning when solving PISA-type problems is not a static internal cognitive capacity, but a dynamically developed discourse. Mathematical reasoning is an essential pillar in mathematics learning that goes beyond mere memorization of formulas or mechanical arithmetic

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skills (Smit et al., 2023) . Viewed from the perspective of a cognitive framework, students' mathematical thinking activities are manifested in real terms through the interaction of four main discursive components, namely word uses, visual mediators, routines, and narratives. The difference in reasoning trajectories shown by the subjects in this study proves that the success of contextual problem solving is not solely determined by mastery of algorithmic formulas but also by students' mathematical reasoning (Purnomo et al., 2022) . Instead, the achievement relies heavily on the cognitive flexibility of students in reorganizing the structure of their discourse and moving from reliance on informal visual routines to mathematically justified formal narrative constructions when faced with non-routine situations (Herbert, 2021).

In the first subject (S3), the evolution of reasoning manifested itself through a gradual transition from visual-empirical routines to structured formal deductive articulation. In the initial phase of solving tasks (a) and (b), the S3 problem-solving process is still dominated by informal routines in the form of direct grid point calculations and shallow visual boundary sketches, resulting in incorrect circumference estimations. This is due to the students' lack of experience when faced with PISA model problems (Yansen et al., 2019). Nevertheless, S3 shows a strong cognitive restructuring capacity when reflecting back on the problem rules. S3 subjects immediately modify visual mediators into precise geometric schemes by taking into account the interval between points and the open boundaries on the front side. Drawing can help students to make it easier to do mathematical reasoning (Sari & Darhim, 2020) . In line with this shift, the use of technical vocabulary (word uses) makes the subject transform maturely towards formal terminology such as intervals, length-width dimensions, and wide ratios. This discursive integration enables S3 to apply the right algebraic routines $K = 2(p + l)$ and a 1:2 wide ratio, so that it succeeds in meeting all mathematical reasoning indicators from the formulation of statements, the submission of rebuttal arguments, the generalization of spatial patterns in Task (c), to the withdrawal of narratives of the final conclusion that is contextually valid in Task (d).

Inversely proportional to this achievement, the discursive analysis of the second subject (S4) revealed the existence of persistent comcognitive barriers due to dependence on mechanical procedures and confusion of mathematical dimensions. The cause is a lack of experience in solving problems such as PISA (Fenanlampir et al.,

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2019). Although S4 is able to make initial statements and memorize formal formulas such as $2p + 2l$, such procedural understanding is isolated from the actual contextual spatial meaning. This is evident in Task (b), where S4 fails to generalize the expansion of the area boundary and instead implements a simple scalar additive routine (24 + 5 = 29 m). This obstacle is further heightened in Tasks (c) and (d), when the subject experiences a shift in the meaning of the concept (word uses breakdown) by equating the physical capacity of 48 participants directly to a linear length of 48 meters, which is then erroneously labeled as the field area and substituted into the circumferential equation $2p + 2l = 48$ without final completion. S4's inability to relate the visual mediator to the concept of fundamental geometry causes his routine to stop and produces incomplete narratives, proving that symbolic mastery without an integrated spatial understanding hinders the formation of valid mathematical reasoning (Anwar & Rahmawati, 2017).

The difference in reasoning dynamics shown by the two subjects confirms how crucial the role of verbal mediation and dialogic probing techniques are in initiating the process of students' cognitive restructuring. Basically, the initial written representations made by students are often still intuitive, fragile, and prone to spatial misconceptions (Mainali, 2021). However, stimuli are in the form of reflective questions given during interviews such as when asking students to evaluate the 24-meter circumference claim by the committee as well as the stage front of the stage rules that have proven to be a catalyst that triggers students' metacognitive awareness (Li et al., 2024). Through this dialogical interaction, both S3 and S4 are encouraged to rearticulate their reasoning structures verbally, identify logical weaknesses in their own calculations, and build tested rebuttal arguments against erroneous concepts of perimeter. These findings reinforce the view that formative interventions and scaffolding play a vital role in guiding students to self-correction, while shifting their mathematical discourse from mere informal everyday communication to one that is more scientific, analytical, and has a higher level of mathematical justification.

CONCLUSION

The findings of the study showed that the first student successfully met all indicators of mathematical reasoning, including the creation of written mathematical

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statements, prediction of answers, preparation of evidence through rebuttal arguments, generalization of spatial relationship patterns, and the drawing of valid conclusions after going through the cognitive restructuring process. On the other hand, the second student only met the indicators of making initial mathematical statements, predicting answers, and rebuttals of concepts through verbal mediation, but failing to meet the indicators of pattern generalization, preparation of proof steps, and drawing conclusions due to dependence on mechanical procedures and confusion of mathematical dimensions. Further research is expected to cover a wider and more heterogeneous group and investigate how learning discourses and dialogical scaffolding strategies can be systematically optimized to facilitate the transformation of students' mathematical reasoning in solving complex contextual problems.

DECLARATION

Phenomenon: Jurnal Pendidikan MIPA was selected as the target journal for the publication of our paper because this journal strives to promote the implementation of high-quality mathematics and science education at all educational levels, an objective that aligns with the aims and scope of this internationally reputable journal. We confirm that both authors have approved the manuscript for publication in your journal.

Availability of Data and Materials	:	The data supporting the study was classified according to the institutions' authorities; therefore, the participants only agreed to participate in the study based on anonymity, and a confidentiality agreement was reached. The data can therefore not be divulged to any third party without official permission granted from the institutions which can either be accepted or declined.
Competing interests	:	The authors declare that they have no conflicting interests in the publication of the paper in your journal.
Funding	:	The present research was sponsored by the authors.

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Authors' contributions	:	Yusuf Arkham Prihandika: Conceptualization, Writing-Original Draft, Editing and Visualization; Wardono: Validation, Review and Formal Analysis; Emi Pujiastuti: Methodology, Validation and Supervision; and Walid: Methodology, Validation and Supervision.
Generative AI Statement	:	AI used for limited, non-substantive support. Generative AI tools, such as Gemini AI, were employed solely for language editing and phrasing enhancements. All conceptualization, analysis, and scholarly content were independently developed and verified by the authors.

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