Phenomenon , 2025, Vol. 15 (No. 2), pp. 240 - 275
Phenomenon : Jurnal Pendidikan MIPA
phenomenon@walisongo.ac.id

Mental Constructions of Male and Female Pre-Service Mathematics Teachers' In Solving Systems of Linear Equations via APOS Theory

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Abstract

Different solution strategies may produce the same correct answer while involving different mathematical constructions. This study addressed the need to examine these constructions and verification practices among male and female pre-service mathematics teachers by integrating Polya's framework with APOS theory. A qualitative explanatory approach involved 38 students who had completed an Elementary Linear Algebra course. Six students were interviewed, and four two male and two female—were purposively selected for obtaining correct answers through distinct strategies. Written solutions and interviews were analyzed using Polya's stages and APOS structures and mechanisms. All participants successfully understood the problem, devised a plan, and carried out their strategies. The male participants used Gauss–Jordan elimination and direct substitution, whereas the female participants incorporated arithmetic-sequence relationships through substitution or the arithmetic-series formula. Beyond these procedural differences, APOS analysis identified different conceptual connections and Schema organizations underlying the same correct solution. All demonstrated Action, Process, Object, and Schema structures through interiorization, coordination, and encapsulation. Within the study's coding framework, reversal was evidenced through procedural rechecking in the male participants, but comparable evidence was not identified in the female participants. Nevertheless, no participant demonstrated complete final verification; evidence of de-encapsulation associated with verification was limited, and thematization was not clearly identified. These findings reveal individual construction pathways and distinguish procedural monitoring from final verification. Instruction should encourage strategy comparison, conceptual explanations, and explicit verification.

Keywords: *G*ender; pre-service mathematics teachers; problem solving; APOS theory; Polya's stages.

Konstruksi Mental Calon Guru Matematika Laki-Laki dan Perempuan dalam Menyelesaikan Sistem Persamaan Linear Berdasarkan Teori APOS

Abstrak

Strategi penyelesaian berbeda dapat menghasilkan jawaban benar yang sama melalui konstruksi matematis yang berbeda. Penelitian ini menjawab kebutuhan untuk mengkaji konstruksi tersebut dan praktik verifikasi pada

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mahasiswa calon guru matematika laki-laki dan perempuan melalui integrasi kerangka Polya dan teori APOS. Penelitian menggunakan pendekatan kualitatif eksplanatori yang melibatkan 38 mahasiswa yang telah menempuh mata kuliah Aljabar Linear Elementer. Enam mahasiswa diwawancarai, kemudian empat yang terdiri dari dua laki-laki dan dua perempuan dipilih secara purposif karena memperoleh jawaban benar melalui strategi berbeda. Pekerjaan tertulis dan wawancara dianalisis berdasarkan tahapan Polya serta struktur dan mekanisme APOS. Semua subjek berhasil memahami masalah, menyusun rencana, dan melaksanakan strategi. Subjek laki-laki menggunakan eliminasi Gauss–Jordan dan substitusi langsung, sedangkan subjek perempuan mengintegrasikan hubungan barisan aritmetika melalui substitusi atau rumus jumlah deret aritmetika. Selain perbedaan prosedur, analisis APOS mengidentifikasi perbedaan koneksi konsep dan organisasi Schema yang mendasari jawaban benar yang sama. Semua subjek menunjukkan struktur Action, Process, Object, dan Schema melalui interiorization, coordination, dan encapsulation. Berdasarkan kerangka pengodean penelitian, reversal ditunjukkan melalui pemeriksaan langkah pada subjek laki-laki, sedangkan bukti sebanding tidak teridentifikasi pada subjek perempuan. Namun, tidak ada subjek yang menunjukkan verifikasi akhir secara lengkap; bukti de-encapsulation terkait verifikasi terbatas, dan thematization belum teridentifikasi secara jelas. Temuan ini mengungkap jalur konstruksi individual serta membedakan pemeriksaan prosedural dari verifikasi akhir. Pembelajaran disarankan mendorong perbandingan strategi, penjelasan konseptual, dan verifikasi eksplisit.

Kata kunci : Gender; mahasiswa calon guru matematika; pemecahan masalah; teori APOS; tahapan Polya.

INTRODUCTION

Mathematical problem solving involves interpreting a situation, choosing a representation, implementing a strategy, and evaluating the result. Research treats these activities as central to mathematics learning and teaching (Artigue & Houdement, 2007; Doorman et al., 2007; Hembree & College, 1992; Liljedahl & Cai, 2021; Santos-Trigo et al., 2021; Santos-Trigo & Reyes-Martínez, 2019) . Pólya's four-stage framework understanding the problem, devising a plan, carrying out the plan, and looking back provides a useful lens for examining what students do. Actual solution processes can include detours and revisions rather than a fixed linear sequence (Rott et al., 2021) . Recent studies of prospective mathematics teachers document differences in problem-solving perceptions, reasoning, and visual strategies (Jiang et al., 2022; Palengka et al., 2022; Zulu & Mudaly, 2023).

For pre-service teachers, solving a problem also has professional relevance: their own representations and explanations may shape how they interpret and discuss

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students' work. Studies of teacher knowledge, discussion practices, and preservice problem solving emphasize the value of attending to varied strategies and the reasoning behind them (Apriani et al., 2021; Biccard, 2020). Other research suggests that teachers need opportunities to respond to incorrect reasoning, select meaningful tasks, and develop their own approaches to nonroutine problems (Beghetto, 2017; Güner & Erbay, 2021; Hallman-Thrasher, 2017). Broader studies of teacher competence and technology-supported strategy exploration provide additional context for such preparation (Santos-Trigo et al., 2021). Studies of algebraic thinking, rich problems, task design, and higher-order problem posing add context for teacher preparation (Gomes et al., 2022; Jailani et al., 2023; Kusuma et al., 2024; A. Putri et al., 2025).

The algebraic context makes the choice of representation especially visible. An equal total and fixed differences between consecutive quantities can be expressed as a system of equations or as an arithmetic sequence. Work on linear algebra teaching has documented the importance of sense-making and of linking procedures to their mathematical meaning (Carlson, 1993). Research on representations, diagrams, visual heuristics, and problem-solving instruction likewise points to the need to examine how a solver organizes the given quantities before calculating (Boonen et al., 2016). A recent Indonesian textbook analysis also identifies the need to connect procedures for systems of linear equations to the ideas that justify them (Agustito et al., 2025). Studies of models and representations among prospective teachers further motivate examining distinct solution paths (Rumite et al., 2023).

Gender provides a further lens for examining how students represent and solve algebraic problems, although differences observed in small groups should not be interpreted as fixed characteristics of male and female learners. In a study involving six mathematics education students solving a three-variable linear equation problem, reported differences across Polya's stages, including in mathematical modelling and checking solutions (Febrilia et al., 2023). By contrast, a study of four junior high school students solving two-variable linear equation problems reported stronger algebraic reasoning among its female participants (Kurniyasih & Nugraheni, 2023). These findings concern different tasks and educational levels; together, they indicate the value of examining the steps and representations used by individual solvers rather than assuming a uniform gender pattern.

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Research involving prospective teachers also points to variation within problem-solving processes. Found differences in the patterns of metacognitive awareness and evaluation observed among male and female participants (Ramlah et al., 2024) . In linear algebra education, showed that approaches emphasizing multiple representations and problems can support preservice teachers' understanding; their study did not test gender differences (Ayyıldız Altınbaş et al., 2025). Building on these distinct lines of research, the present study examines how two male and two female preservice mathematics teachers represent the same system-of-linear-equations task, carry out their chosen strategies, and document any verification. Its purpose is to describe these four cases, not to attribute a strategy or level of ability to gender.

Strategic flexibility does not end when an answer has been obtained. Looking back may involve checking the original conditions, explaining why a method works, or comparing it with another method. Research on metacognition, self-monitoring, and productive reflection provides reasons to distinguish an accurate calculation from a justified solution (Radmehr & Drake, 2017; Schneider & Artelt, 2010; Tachie, 2019). Studies of multiple strategies and classroom discussion further suggest that comparing approaches can expose mathematical connections that remain hidden when solutions are considered separately (Eisenmann et al., 2015; Hourigan & Leavy, 2023; Santos-Trigo & Reyes-Martínez, 2019). Metacognitive instruction and peer assessment provide further contexts for examining reflection and justification in prospective teachers' mathematical work (de-Armas-González et al., 2023; Prabawanto, 2023).

Polya's framework provides a useful description of what solvers do when understanding a problem, planning a solution, implementing a strategy, and evaluating the result. However, identifying these stages and the final answer does not, by itself, explain how solvers construct and connect the mathematical knowledge underlying their solutions. APOS theory offers a complementary perspective by examining Action, Process, Object, and Schema structures and the mechanisms through which they develop. Integrating these frameworks therefore enables an analysis of both the problem-solving activities and the mathematical constructions underlying them (Arnon et al., n.d.).

Previous studies have examined mathematical problem solving from various perspectives. (Yayuk & Husamah, 2020) analyzed the difficulties experienced by

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prospective elementary school teachers in solving mathematical problems using Polya's four stages. Their findings revealed difficulties across all four stages, including connecting concepts, applying knowledge, and checking answers. Similarly, (Sumarni & Darhim, 2021) investigated prospective mathematics teachers' problem-solving abilities using Polya's stages. They identified difficulties in understanding problems and performing arithmetic operations and found that none of the participants checked their solutions. These studies provide insights into the difficulties prospective teachers encounter at different stages of problem solving. (Dewi et al., 2024) examined prospective mathematics teachers' conceptual understanding when determining solutions to systems of linear equations (SLEs). Their findings indicated that some participants continued to rely on memorized properties and procedures and experienced difficulties connecting algorithms to their underlying concepts. Meanwhile, (Jiang et al., 2022) investigated prospective mathematics teachers' perceptions of mathematical problem solving and its teaching. Their study highlighted the importance of exploration, sustained thinking, and metacognitive engagement in problem-solving instruction. Accordingly, research on mathematical problem solving should consider both solution activities and the conceptual understanding and thinking underlying those activities.

(Lerebulan & Ratuanik, 2023) investigated upper secondary school students' conceptual understanding of systems of linear equations in two variables through the APOS constructions of action, process, object, and schema, with attention to students' learning styles. (Yunita et al., 2024) analyzed lower secondary school students' problem solving involving systems of linear equations in two variables using APOS theory, considering differences in mathematical ability. Their findings showed that students with high mathematical ability exhibited action, process, object, and schema constructions, whereas those with moderate mathematical ability exhibited action, process, and object constructions, and those with low mathematical ability exhibited only action constructions. These studies demonstrate the usefulness of APOS theory in identifying mental constructions manifested in students' understanding and solution of algebraic problems. Their findings revealed variations in the APOS constructions identified in students' solutions. Meanwhile, (Syaiful et al., 2025) investigated the implementation of APOS-based and modified APOS-based instruction to improve university students' mathematical critical thinking skills.

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Examining these constructions is relevant to mathematics teacher education because understanding prospective teachers' reasoning requires attention to the mental organization underlying their solution procedures. (Rumite et al., 2023) Identified different mental models associated with different strategies and schemas, while (Prabawanto, 2023) Demonstrated that metacognitive instruction improved prospective mathematics teachers' reversible thinking. Together, these findings provide a rationale for investigating how prospective teachers organize mathematical knowledge and reconsider their reasoning during problem solving.

Although these studies contribute to understanding problem-solving processes and outcomes, they have not specifically focused on the relationships between activities at each of Polya's stages and the underlying APOS mental structures and mechanisms among male and female prospective mathematics teachers solving the same contextual SLE problem. To address this gap, the present study integrates Polya's problem-solving framework with APOS theory to reveal similarities and differences in the mental constructions underlying participants' solution strategies. This integration enables an examination of problem-solving activities alongside the mental structures and mechanisms underlying the construction and coordination of mathematical concepts throughout the solution process.

The findings of this analysis are expected to provide a basis for educators to consider appropriate forms of instructional support or intervention for prospective mathematics teachers who experience difficulties interpreting contextual information, constructing mathematical models, connecting concepts and representations, implementing solution procedures, and verifying results. Understanding the mental constructions underlying these activities may help educators provide more targeted support, including guiding questions, activities that strengthen conceptual connections, comparisons of solution strategies, and routine opportunities to verify results, according to individual students' needs. Therefore, this study aims to describe and compare the mental constructions of male and female prospective mathematics teachers as they solve a contextual SLE problem at each of Polya's stages through the lens of APOS theory. The findings are intended to inform the design of instructional support that addresses students' difficulties and needs, without assuming that any observed differences are determined by gender.

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RESEARCH METHODS

This study used a qualitative, explanatory design to investigate the problem-solving processes of male and female pre-service mathematics teachers as they solved a contextual system-of-linear-equations problem. The analysis integrated Polya's problem-solving framework and APOS theory**.** Polya's four stages—understanding the problem, devising a plan, carrying out the plan, and looking back—were used to trace the participants' problem-solving processes, while APOS theory was used to identify the mental structures and mechanisms underlying these processes. The APOS analysis focused on the mental structures of Action, Process, Object, and Schema and the mental mechanisms of interiorization, coordination, reversal, encapsulation, de-encapsulation, and thematization. Thus, Polya's framework was used to describe the progression of problem solving, whereas APOS theory provided a deeper analysis of how mathematical knowledge was constructed, connected, transformed, and organized during the solution process.

The study was conducted in the Mathematics Education Study Program at Satya Wacana Christian University and involved 38 pre-service mathematics teachers who had completed an Elementary Linear Algebra course. The participants consisted of 10 men and 28 women aged 17–20 years. All participants completed a problem-solving test on systems of linear equations. Six participants—three men and three women—were subsequently interviewed, and four participants were purposively selected for in-depth analysis because they obtained correct solutions using different strategies. The two male participants were coded S1 and S2, while the two female participants were coded S3 and S4. Gender was used to descriptively compare the problem-solving and mental-construction patterns identified in the selected cases rather than to establish general differences in mathematical ability between male and female students. The participant-selection procedure is consistent with that reported in the original study design.

Data were collected through a problem-solving test, participants' written work, video recordings, in-depth interviews, and field notes. The test consisted of one contextual problem developed according to the problem-solving indicators for systems of linear equations, as presented in Figure 1. The interview questions were organized according to Polya's four stages and were used to clarify participants' thinking that was not fully evident from their written solutions. For example, participants were asked to

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identify the given and required information, explain their solution plans and chosen strategies, describe how they carried out the plans, and explain whether and how they checked their solutions. The interviews therefore provided additional evidence for identifying the APOS mental structures and mechanisms that emerged during problem solving. In the original procedure, the interviews were also used to clarify participants' thinking and support the coding process.

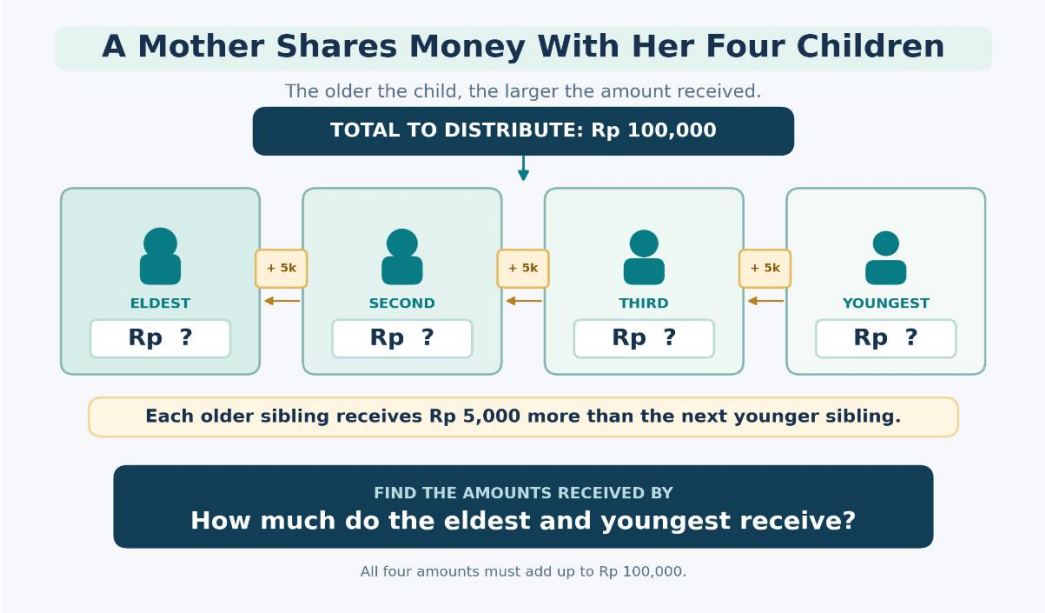


Figure 1. Problem-Solving Test Instrument

Based on Figure 1, it can be observed that the task requires participants to determine the youngest and eldest children's shares by coordinating a total of IDR 100,000, four recipients, and a constant difference of IDR 5,000 between consecutive shares. These conditions allow the same problem to be represented through linear equations or arithmetic-sequence relationships.

Data analysis was conducted in two interconnected stages. First, participants' written work and interview responses were analyzed according to Polya's four problem-solving stages. Second, the thinking processes identified within and across these stages were analyzed using APOS theory. Action represented explicit mathematical actions performed in response to the problem; Process represented the internalization and organization of mathematical operations; Object represented a process conceived as a complete mathematical entity that could be further transformed; and Schema represented an organized network of related mathematical concepts, processes, and objects. The analysis also identified interiorization when Actions developed into

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Processes, coordination when mathematical processes or representations were connected, reversal when participants retraced or reconsidered previous processes, encapsulation when a Process was conceived as an Object, de-encapsulation when an Object was unpacked into its underlying Process, and thematization when a Schema was integrated into a broader conceptual structure. The operational indicators used in the analysis are presented in Table 1.

Table 1: Indicators of Problem-Solving Processes Based on Polya's Framework and APOS Theory.

Polya's Problem-Solving Stage	APOS Mental Structure	Code	Research Indicator	Mental Mechanism
Understanding the Problem	Action	ACT1	Identifies the information given in the problem and what is required.	—
	Action	ACT2	Identifies quantities, variables, and mathematical relationships contained in the problem.	**Interiorization (Int)**
	Action	ACT3	Represents the contextual information in mathematical form, such as equations, variable relationships, matrices, or arithmetic-sequence relationships.	**Interiorization (Int)**
Devising a Plan	Process	PRO1	Recalls and explains mathematical concepts relevant to the problem, such as systems of linear equations, substitution, matrix operations, or arithmetic sequences/series.	**Interiorization (Int)**
	Process	PRO2	Mentally organizes a sequence of operations and selects an appropriate strategy for solving the problem.	**Coordination (Coor)**
	Process	PRO3	Explains how the planned mathematical transformations will lead to the required solution.	**Coordination (Coor)**
Carrying Out the Plan	Process/Coordination	COO1	Connects relevant mathematical processes and representations to implement the selected strategy.	**Coordination (Coor)**
	Process/Coordination	COO2	Coordinates mathematical relationships, operations, and representations to obtain the solution.	**Coordination (Coor)**
	Process	REV1	Retraces or reverses previously performed processes when checking,	**Reversal (Rev)**

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			reconsidering, or correcting a solution step.	
	Object	OBJ1	Conceives the mathematical process or representation as a whole mathematical entity that can be operated on or transformed.	**Encapsulation (Enc)**
	Object	OBJ2	Flexibly operates on or transforms the mathematical object to obtain or interpret the solution.	**Encapsulation (Enc)**
Looking Back	Object/Process	DEC1	Unpacks the mathematical object into its underlying processes to explain or verify how the solution was obtained.	**De-encapsulation (De-enc)**
Integration Across Problem-Solving Stages	Schema	SCH1	Organizes relevant concepts, procedures, representations, and relationships into a coherent mathematical structure.	—
	Schema	SCH2	Selects and uses an appropriate strategy based on the characteristics of the problem and relationships among mathematical concepts.	—
	Schema	SCH3	Integrates the problem representation, solution process, result, and verification into a coherent problem-solving structure.	—
Extended Conceptual Integration	Schema/Object	THE1	Connects the constructed schema with a broader network of mathematical concepts or treats the schema as an entity for further mathematical reasoning.	**Thematization (Them)**

Based on Table 1, it can be observed that the analytical indicators connect Polya's problem-solving activities with APOS mental structures and mechanisms. The table provides operational criteria for identifying how participants represent information, organize and coordinate Processes, treat mathematical constructions as Objects, and integrate their knowledge into Schemas. These criteria guide the interpretation of written solutions and interviews rather than assuming that every participant demonstrates every mechanism.

The indicators in Table 1 were used to code evidence from participants' written solutions and interviews. The resulting codes were then used to construct individual problem-solving profiles for S1–S4 and to descriptively compare the mental structures,

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mental mechanisms, representations, and solution strategies identified across the male and female participants. A structure or mechanism was reported only when supported by evidence from the available data; therefore, the absence of a particular APOS mechanism was interpreted as not identified in the data, rather than as evidence that the participant was incapable of constructing it.

The problem-solving test and interview guide were validated through expert judgment by three lecturers from the Elementary Linear Algebra teaching team. A pilot study was also conducted to obtain empirical evidence concerning the test instrument. The original methodological documentation similarly reports expert validation of both the test and interview instruments.

RESULTS AND DISCUSSION

RESULTS

Four participants were selected for detailed analysis: two male participants (S1 and S2) and two female participants (S3 and S4). All four correctly determined that the youngest child received IDR 17,500 and the eldest received IDR 32,500. Their written work and interviews were examined using Polya's stages of understanding the problem, devising a plan, carrying out the plan, and looking back.

The written solutions and interview data of the four participants were analyzed using Polya's problem-solving stages integrated with the APOS framework. Polya's stages were used to trace the progression of each participant's problem-solving process, while APOS theory was used to identify the mental structures and mechanisms emerging within and across these stages. The analysis focused on the mental structures of Action, Process, Object, and Schema and on the mechanisms of interiorization, coordination, reversal, encapsulation, de-encapsulation, and thematization. The findings are presented for each participant to illustrate how different solution strategies were associated with different mathematical constructions.

APOS analysis and problem-solving process of the Male Participants

Participant 1 (S1): Gauss–Jordan elimination.

Participant 1 (S1) obtained the correct answer using Gauss–Jordan elimination. The written solution is shown in Figure 3.

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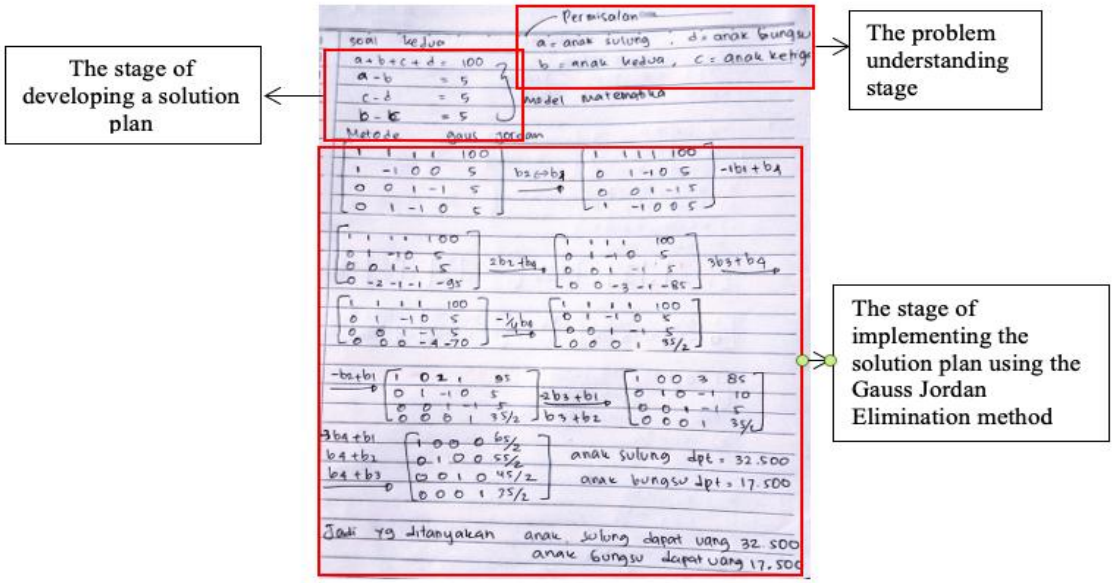


Figure 3. Participant 1's problem-solving process.

Based on Figure 3, it can be observed that S1 translates the contextual relationships into a system of linear equations and uses Gauss–Jordan elimination to obtain the solution. The written work provides evidence of how the equation representation is transformed into a matrix-based procedure.

Participant 1 (S1) solved the problem using Gauss–Jordan elimination. Analysis of the written solution and interview data showed the development of Action, Process, Object, and Schema structures through several APOS mental mechanisms. During the understanding-the-problem stage, S1 identified the total amount of IDR 100,000, the four children, the difference of IDR 5,000 between consecutive shares, and the amounts to be determined (ACT1). S1 then represented the four shares using the variables (a, b, c,) and (d) and constructed a system of linear equations (ACT2–ACT3). These Actions provided the basis for the interiorization of the contextual relationships into mathematical Processes.

R: *What information is given in the problem?*
S1: *A mother intends to distribute 100,000 among her four children. Each older child receives 5,000 more than the next younger child.*
R: *What does the problem ask you to find?*
S1: *How much money do the eldest and youngest children receive?*
R: *All right. Let us look at your work. Could you explain how you solved the problem?*
S1: *First, I let a represent the eldest child's share, b the second child's, c the third child's, and d the youngest child's. The total amount they receive is 100,000. I wrote it this way, Ma'am, and divided by 1,000 to make the calculations easier.* (Points to their written work.)
R: *Please explain what you did next.*
S1: *All right, Ma'am. Since the difference between the amounts received by children adjacent in age is 5,000, I divided that by 1,000 as well to make the calculations easier. I then wrote the equations like this. Oh, I forgot one more equation.* (Points to their written work.)

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At the Process level, S1 recalled the relationship between a system of linear equations, an augmented matrix, and Gauss–Jordan elimination (PRO1). The participant planned to transform the system into an augmented matrix and apply elementary row operations systematically until reduced row echelon form was obtained (PRO2). The plan was then carried out through row interchanges, elimination, and scaling operations to determine the values of the four variables (PRO3). This process demonstrated coordination between the system of equations, its matrix representation, elementary row operations, and the resulting numerical solution (COO1–COO2).

R: *How did you approach solving the problem?*
S1: *I chose the Gauss–Jordan elimination method, Ma'am.*
R: *Why did you choose Gauss–Jordan elimination?*
S1: *I thought its steps were systematic, Ma'am. I did not have to decide which equation to eliminate as I did in high school. I was also more comfortable using Gauss–Jordan elimination because, unlike Gaussian elimination, it does not require back substitution. That is why I chose it.*

A reversal mechanism (REV1) was also identified during the execution process. S1 stated, "I recheck the steps I have completed before proceeding to the next step." This indicates that the participant mentally retraced previously performed operations before continuing with subsequent operations. However, this procedural rechecking did not extend to a complete final verification by substituting the obtained values into the original equations.

R: *What do you do to ensure that you do not make any computational errors?*
S1: *I recheck the steps I have completed before proceeding to the next step, Ma'am.*
R: *All right. Are you confident in your answer? Would you consider using a different solution?*
S1: *Yes, Ma'am. I am confident in my answer.*
R: *I will give you an opportunity to review your work once more. Are you completely confident in your answer?*
S: *Yes, Ma'am. I have reviewed it, and I am confident in my answer.*

The repeated coordination of the system, augmented matrix, and row operations indicates that S1 was able to conceive of the matrix representation and its transformations as an integrated mathematical object (OBJ1–OBJ2). The participant subsequently organized the system of linear equations, matrix representation, Gauss–Jordan procedure, and resulting solution into a coherent schema (SCH1–SCH2). However, de-encapsulation was only partially evidenced because the final solution was not unpacked and verified against the original system (DEC1). Similarly, complete

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integration of the solution and its verification (SCH3) and broader thematization (THE1) were not clearly identified in the available data.

Overall, S1's problem-solving process showed a progression from Action to Process, Object, and Schema, primarily through interiorization, coordination, reversal, and encapsulation. The participant obtained the correct values of IDR 32,500 for the eldest child and IDR 17,500 for the youngest child, although final verification against the original equations was not demonstrated. APOS analysis and problem-solving process of Participant 1 (S1) is presented in Figure 4.

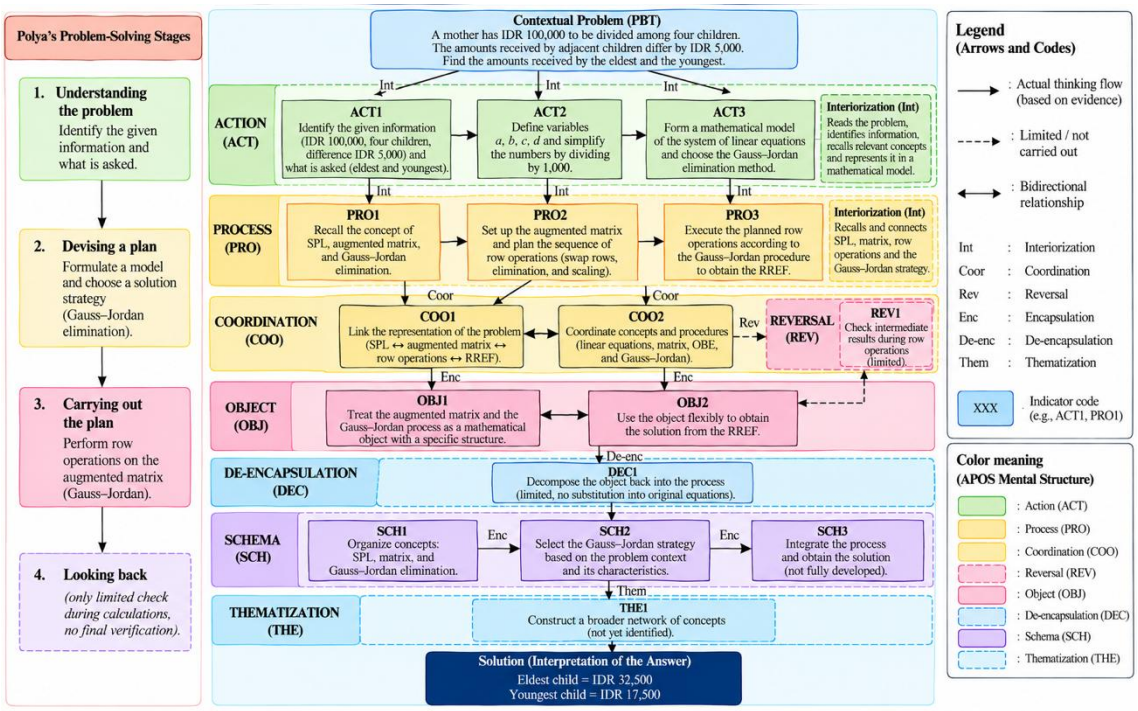


Figure 4. APOS analysis and problem-solving process of Participant 1 (S1)

Based on Figure 4, it can be observed that S1's solution pathway connects contextual information, linear equations, an augmented matrix, and elementary row operations. This organization illustrates the APOS constructions identified in the case and provides a basis for distinguishing the reported rechecking of preceding steps from complete verification against the original conditions.

Participant 2 (S2) : Substitution.

Participant 2 obtained the correct answer using substitution. The written solution is shown in Figure 5.

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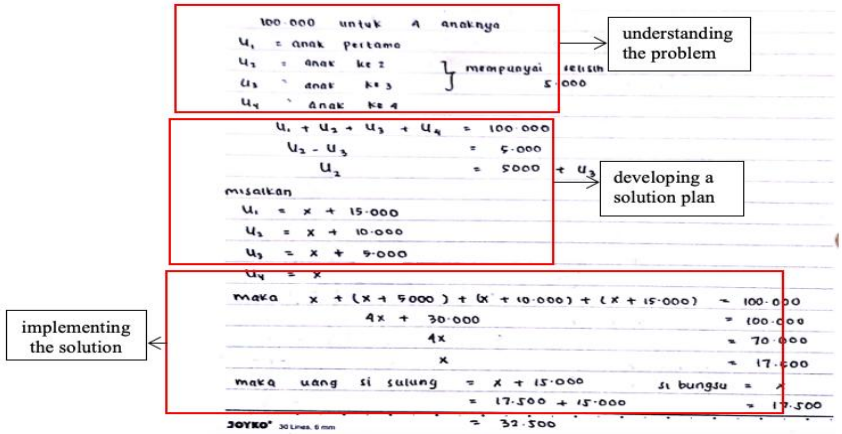


Figure 5. Participant 2's problem-solving process.

Based on Figure 5, it can be observed that S2 expresses the children's shares in terms of one unknown and substitutes these expressions into the total-amount equation. The solution therefore depends on coordinating variable relationships within a direct substitution procedure.

Participant 2 (S2) solved the problem using substitution. At the Action level, S2 identified the given and required information (ACT1), represented the four children's shares using variables (ACT2), and expressed each share in relation to the youngest child's share x (ACT3). Through interiorization, these explicit relationships developed into a Process in which S2 conceived the shares as expressions in one unknown.

R: *What information is given in the problem?*
S2: *An amount of 100,000 is to be distributed among four children. Each older child receives 5,000 more than the next younger child.*
R: *What does the problem ask you to determine?*
S2: *How much money do the eldest and youngest children receive?*
R: *All right. Let us examine your work. Could you explain how you solved the problem?*
S2: *First, I denoted the amounts received by the first, second, third, and fourth children as u_1, u_2, u_3, and u_4, respectively. The total amount they receive is 100,000, so I wrote an equation for their combined shares. Because the difference between the amounts received by children adjacent in age is 5,000, I then expressed each child's share in terms of that difference. I then decided to let x represent the amount received by the fourth child.*

At the Process level, S2 recalled substitution as an appropriate method (PRO1), planned to express all four shares in terms of x and substitute them into the total-amount equation (PRO2), and executed the substitutions to obtain $x = 17{,}500$ and the eldest child's share of 32,500 (PRO3). Coordination occurred as S2 connected the relationships among the four variables with the total-amount equation and the substitution procedure (COO1–COO2).

R: *So, which method did you use to solve this problem?*
S2: *I used the substitution method, Ma'am.*

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Reversal (REV1) was identified when S2 explained that the result obtained at each preceding step was rechecked before proceeding to a new step. Nevertheless, S2 declined to perform an additional final check. Thus, as in S2, reversal occurred within the solution process, whereas final verification of the complete solution was not demonstrated.

R: *Are you confident in your answer?*
S2: *Yes, Ma'am. I am confident in my answer.*
R: *Do you not want to check your answer again?*
S2: *No, Ma'am. I do not think I need to check it again.*
R: *What makes you confident in your answer?*
S2: *Before proceeding to each new step, I always recheck the result I obtained in the previous step.*
R: *Are there any other methods that could be used to solve this problem?*
S2: *Yes, Ma'am. It could also be solved using the elimination method, Gaussian elimination, or Gauss–Jordan elimination.*

APOS analysis and problem-solving process of Participant 2 (S2) is presented in Figure 7.

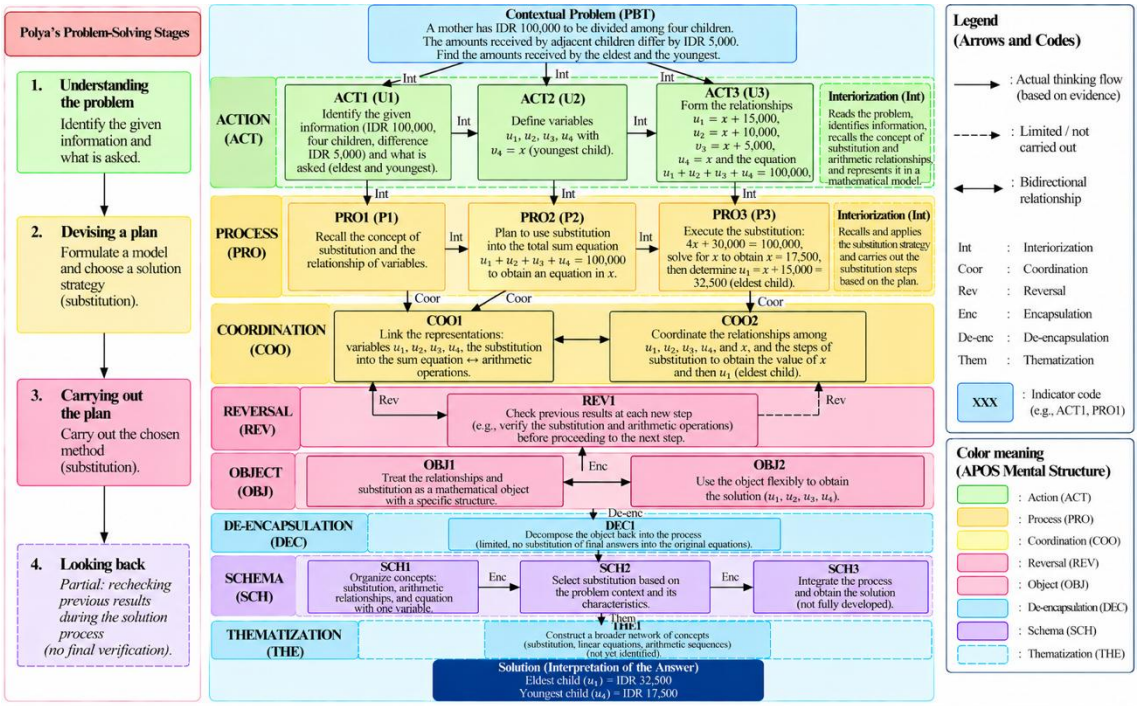


Figure 7. APOS analysis and problem-solving process of Participant 2 (S2)

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Based on Figure 7, it can be observed that S2's APOS pathway is organized around the relationships among the four shares and the substitution Process. The analysis distinguishes this organization from a solution that explicitly coordinates arithmetic-sequence concepts with substitution, even though both approaches lead to the same numerical answer.

APOS analysis and problem-solving process of the Female Participants
Participant 3 (S3): arithmetic-sequence representation and substitution.

Participant 3 obtained the correct answer using substitution and an arithmetic-sequence representation. The written solution is shown in Figure 8.

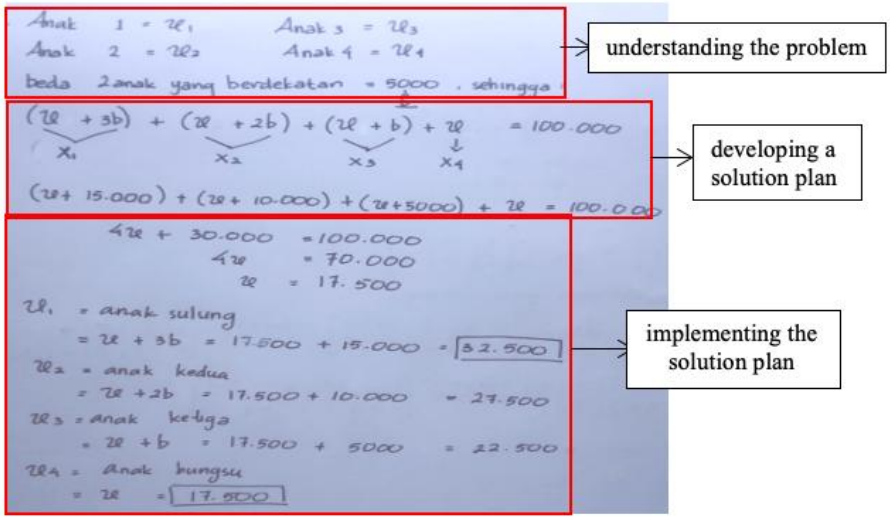


Figure 8. Participant 3's problem-solving process.

Based on Figure 8, it can be observed that S3 uses the constant difference to represent the shares as an arithmetic sequence before applying substitution. The written solution makes the connection between arithmetic-sequence relationships and a linear equation visible. Participant 3 (S3) combined an arithmetic-sequence representation with substitution. At the Action level, S3 identified the problem information (ACT1), represented the youngest child's share by x and the common difference by $b = 5000$ (ACT2), and expressed the remaining shares as $x + 3b, x + 2b, x + b$ and x (ACT3). These representations were interiorized into Processes involving both arithmetic-sequence relationships and substitution.

R: *What information do you think is given in the problem?*
S3: *A mother has 100,000 to distribute among her four children. The difference between the amounts received by children adjacent in age is 5,000, with each older child receiving more than the next younger child.*
R: *What does the problem ask you to determine?*
S3: *How much money do the eldest and youngest children receive?*

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At the Process level, S3 recalled the concepts of arithmetic sequences and substitution (PRO1), planned to express the four shares using the common difference and substitute them into the total equation (PRO2), and then carried out the substitution to determine the youngest and eldest shares (PRO3). Coordination was particularly evident because S3 connected the arithmetic-sequence representation with a linear equation and subsequently with the substitution procedure (COO1–COO2). The interview further showed that S3 initially intended to use the arithmetic-sequence concept but, after reconsidering the equation, recognized that it could be solved directly by substitution.

R: *All right. Let us examine your work. Could you explain how you solved the problem?*
S3: *First, I assigned a variable to the amount received by each child: the first* x_1*, second* x_2*, third* x_3*, and fourth child* x_4*, respectively. Because the difference between the amounts received by two children adjacent in age is 5,000, I set* $b = 5{,}000$*. I also assigned a variable to the amount received by the fourth child. The total amount distributed is 100,000, so I wrote an equation for the sum of their shares. Then,* $x + 3b + x + 2b + x + b + x = 100{,}000$
R: *So, which method did you use to solve this problem?*
S3: *I used substitution, Ma'am.*
R: *Based on your written work and explanation, could you have solved the equation without assigning b to the value 5,000?*
S3: *Yes, Ma'am. I initially intended to use the concept of an arithmetic sequence. However, when I looked at the equation again, I realized that I could solve it directly, so I returned to the substitution method.*

No clear evidence of the reversal mechanism (REV1) was identified in S3's data. Although the participant reconsidered the initially intended strategy, the available evidence does not demonstrate retracing or reversing a completed mathematical process for verification. Final de-encapsulation was also limited because the obtained values were not substituted back into the original conditions.

R: *Are you confident in your answer?*
S3: *Yes, Ma'am, I am.*
R: *Would you like to check your work again?*
S3: *No, Ma'am.*

APOS analysis and problem-solving process of Participant 3 (S3) is presented in Figure 9.

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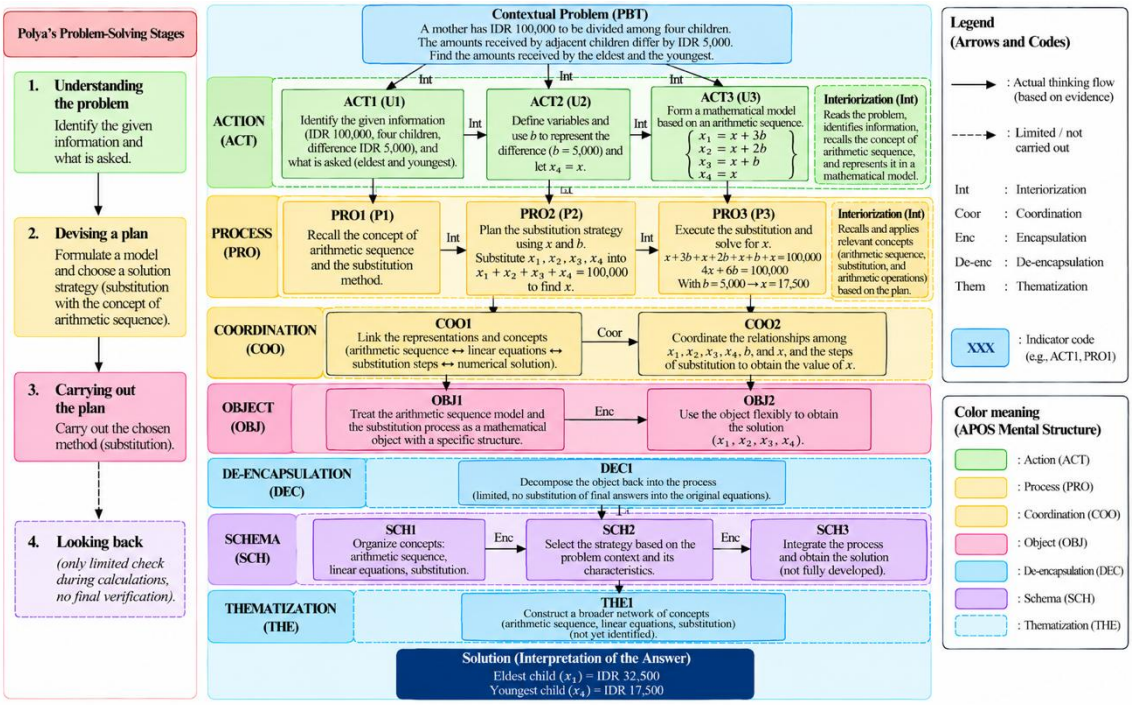


Figure 9. APOS analysis and problem-solving process of Participant 3 (S3)

Based on Figure 9, it can be observed that S3's solution coordinates arithmetic-sequence relationships with linear equations and substitution. This connection supports a Schema that incorporates both sequence concepts and algebraic procedures; comparable evidence of procedural reversal was not identified in the available data.

Participant 4 (S4): Arithmetic series formula.

Participant 4 obtained the correct answer using an arithmetic series formula. The written solution is shown in Figure 10.

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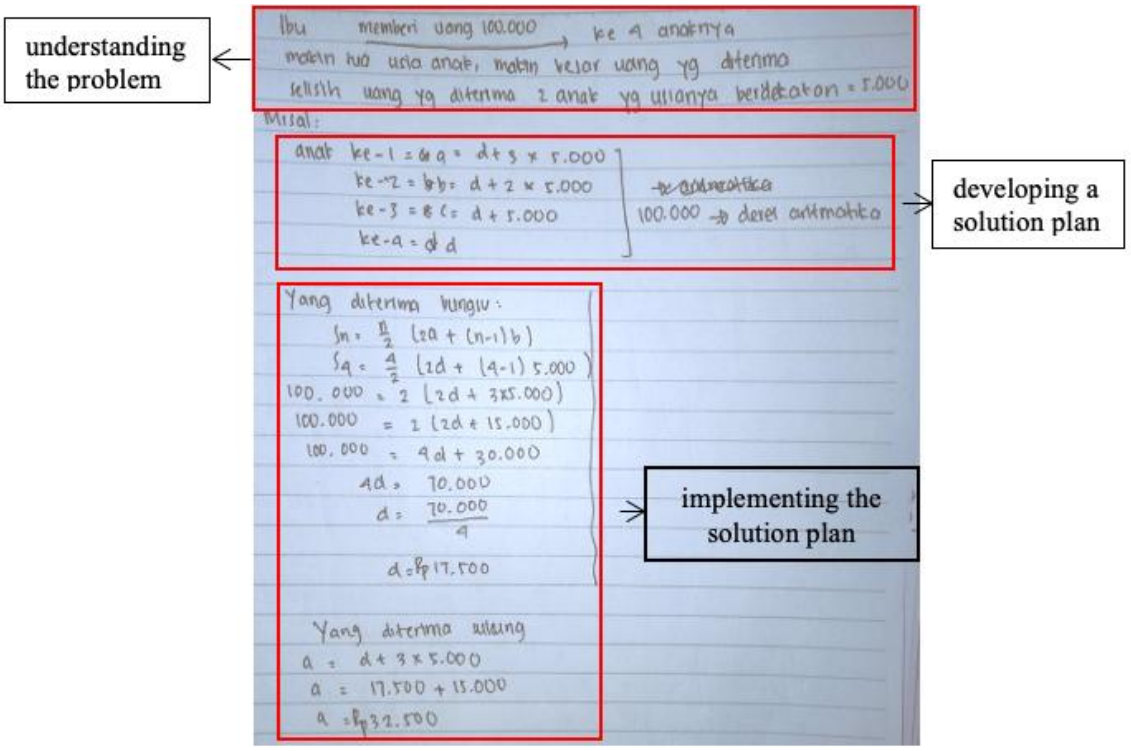


Figure 10. Participant 4's problem-solving process.

Based on Figure 10, it can be observed that S4 organizes the four shares using an arithmetic-sequence structure and applies the arithmetic-series formula to the known total. This strategy connects the number of children, the common difference, and the sum to determine the required shares.

Participant 4 (S4) constructed the solution primarily through the arithmetic-sequence and arithmetic-series structures. At the Action level, S4 identified the given and required information (ACT1), represented the four children's shares using variables (ACT2), and expressed their relationships using a constant difference of IDR 5,000 (ACT3).

> ***R:*** *What information do you think is given in the problem?*
> ***S4:*** *An amount of 100,000 is to be distributed among four children. Each older child receives more money than the next younger child, with a difference of 5,000 between their shares.*
> ***R:*** *What does the problem ask you to determine?*
> ***S4:*** *How much money do the eldest and youngest children receive?*

These Actions were interiorized into a Process in which the four shares were conceived as terms of an arithmetic sequence. S4 recalled the arithmetic-series formula (PRO1), planned to use the known number of terms, common difference, and total sum to determine the youngest child's share (PRO2), and implemented the formula to obtain $d = 17{,}500$, followed by $a = 32{,}500$, (PRO3). Coordination occurred between the

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arithmetic-sequence representation, the known quantities, and the arithmetic-series formula (COO1–COO2). Through encapsulation, these related Processes were treated as an integrated mathematical Object that could be used to determine the unknown shares (OBJ1–OBJ2). The participant subsequently organized the relevant arithmetic-sequence and series concepts into a Schema appropriate to the problem (SCH1–SCH2).

> ***R:*** *All right. Let us examine your written work. Could you explain how you solved the problem?*
> ***S4:*** *First, I denoted the amounts received by the first, second, third, and fourth children as a, b, c, and d, respectively. Since the difference between the amounts received by children adjacent in age is 5,000, I let the fourth child's share be d. The third child's share is then c=d+5,000, the second child's share is b = d+2(5,000), and the first child's share is a = d+3(5,000).*
> ***R:*** *So, which method did you use to solve this problem?*
> ***S4:*** *I used the arithmetic series formula, Ma'am, because all the required information was available: the fourth term, the common difference, and the number of terms.*

No clear evidence of reversal was identified, and de-encapsulation remained limited because S4 did not return to the original conditions to verify that the four shares summed to IDR 100,000 and differed consecutively by IDR 5,000. Consequently, complete integration involving final verification (SCH3) and broader thematization (THE1) was not clearly evidenced in the available data

> ***R:*** *Are you confident in your answer?*
> ***S4:*** *Yes, Ma'am, I am confident in it.*
> ***R:*** *Would you like to check your work again?*
> ***S4:*** *No, Ma'am.*

APOS analysis and problem-solving process of Participant 4 (S4) is presented in Figure 11.

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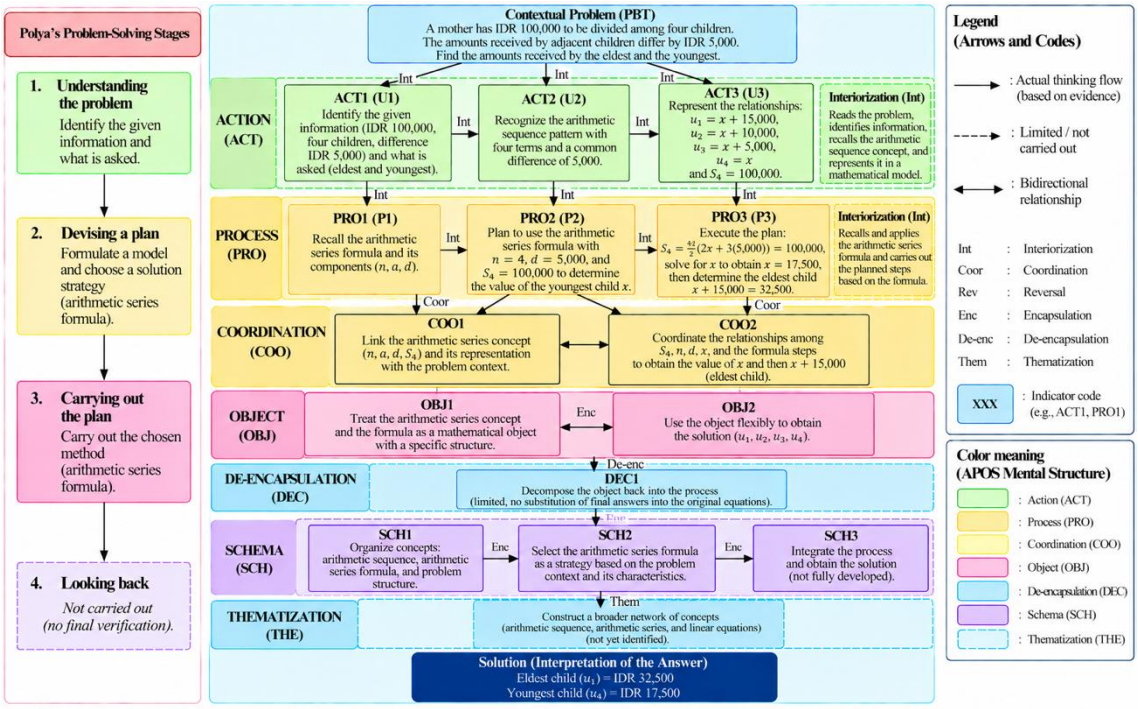


Figure 11. The problem-solving process of Participant 4 (S4)

Based on Figure 11, it can be observed that S4's solution pathway is organized around arithmetic-sequence relationships and the arithmetic-series formula. The APOS analysis describes the conceptual organization supporting this strategy, while the available work and interview do not document complete final verification against all original conditions.

Across the four cases, all participants demonstrated evidence of Action, Process, Object, and Schema structures, although the mathematical constructions through which these structures emerged differed according to their selected strategies. S1 and S2, the male participants, constructed their solutions through Gauss–Jordan elimination and direct substitution, respectively, whereas S3 and S4, the female participants, explicitly incorporated arithmetic-sequence relationships into their solution processes. Interiorization, coordination, and encapsulation were identified across the four cases. Reversal was explicitly evidenced in S1 and S2 through their reported rechecking of preceding steps, whereas comparable evidence was not identified for S3 and S4. None of the four participants demonstrated complete final verification against the original problem conditions; consequently, evidence of de-encapsulation associated with final verification was limited. These patterns characterize the four purposively selected cases and should not be interpreted as general differences attributable to gender.

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DISCUSSION

The findings demonstrate that the same contextual system-of-linear-equations problem supported different mathematical representations and solution strategies. S1 represented the relationships as a system of linear equations and used an augmented matrix and Gauss–Jordan elimination; S2 expressed the four shares in terms of one unknown and used direct substitution; S3 combined an arithmetic-sequence representation with substitution; and S4 organized the quantities as an arithmetic sequence and applied the arithmetic-series formula. Although all four participants obtained the same numerical solution, they organized and transformed the mathematical information differently. This finding highlights the importance of considering multiple representations in mathematical problem solving because different representations may provide different ways of accessing and connecting mathematical relationships. In linear algebra education specifically, showed that instruction involving multiple representations and problem-based approaches can support pre-service teachers' mathematical understanding. The present findings extend this perspective by showing that individual pre-service teachers may spontaneously construct different representations when solving the same contextual problem (Ayyıldız Altınbaş et al., 2025; Ramlah et al., 2024) .

From the perspective of APOS theory, the different solution strategies reflected different pathways through which mathematical knowledge was constructed and organized. All four participants demonstrated evidence of Action when they identified relevant information, defined variables, and translated the contextual situation into mathematical expressions. These Actions were subsequently interiorized into Processes as the participants organized mathematical operations without relying solely on externally directed steps. S1 developed a Process involving a system of linear equations, an augmented matrix, and row operations; S2 developed a substitution Process based on relationships among the four shares; S3 connected arithmetic-sequence relationships with substitution; and S4 organized the quantities through an arithmetic-series Process. In APOS theory, mathematical understanding is characterized through the mental structures of Action, Process, Object, and Schema, with mechanisms such as interiorization and coordination explaining the development and relationships among these structures (Arnon et al., 2014).

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The mechanism of interiorization was evident as the participants moved from explicit Actions to mentally organized Processes. For example, S1 did not merely write a system of equations but recognized that it could be represented as an augmented matrix and transformed through elementary row operations. Similarly, S2 transformed the relationships among the four children's shares into expressions involving a single unknown. For S3 and S4, the constant difference of IDR 5,000 was no longer treated merely as numerical information but became part of an arithmetic-sequence structure used to organize the solution. According to APOS theory, interiorization occurs when externally performed Actions become internal Processes that can be mentally conceived and controlled. Consequently, the evidence in the present study suggests that the participants progressed beyond carrying out isolated operations toward organizing those operations as meaningful mathematical Processes (Arnon et al., 2014).

The findings also reveal the importance of coordination in the participants' problem-solving processes. S1 coordinated the system of linear equations with its augmented-matrix representation, elementary row operations, and the resulting solution. S2 coordinated relationships among variables with substitution and the total-amount equation. S3 coordinated the arithmetic-sequence representation with a linear equation and substitution, whereas S4 coordinated the arithmetic-sequence structure with the arithmetic-series formula. From an APOS perspective, coordination involves connecting Processes in order to construct a new Process, and these coordinated Processes can subsequently contribute to the construction of more integrated mathematical structures. Thus, the participants' successful solutions were not based simply on the execution of procedures but on their ability to connect mathematical relationships, representations, and procedures within their selected strategies (Arnon et al., 2014).

The distinction between S2 and S3 illustrates why examining only the final solution method may obscure important differences in mathematical thinking. Both participants ultimately substituted expressions into an equation representing the total amount; however, S2 directly expressed all shares in terms of one unknown, whereas S3 first constructed an arithmetic-sequence representation using the constant difference before carrying out substitution. S3's solution therefore involved coordination between two conceptual resources—arithmetic sequences and linear equations—before the numerical solution was obtained. This finding is consistent with research emphasizing

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that mathematical problem solving involves not only the application of procedures but also the organization, regulation, and connection of mathematical information. Research on prospective teachers has similarly demonstrated variation in how individuals organize and regulate their mathematical thinking even when working within comparable problem-solving contexts (Ramlah et al., 2024)

The participants also demonstrated evidence of emerging Schemas, although the Schemas were organized differently according to the strategies they selected. S1 organized concepts related to systems of linear equations, augmented matrices, elementary row operations, and Gauss–Jordan elimination. S2 organized variable relationships, linear equations, and substitution. S3 constructed a Schema connecting arithmetic sequences, linear equations, and substitution, whereas S4 connected arithmetic-sequence relationships with the arithmetic-series formula. APOS theory defines a Schema as a coherent organization of Actions, Processes, Objects, and other Schemas that an individual can activate in response to a mathematical situation. The different Schemas observed in the present cases therefore illustrate how participants can organize different networks of mathematical knowledge to solve the same problem successfully (Arnon et al., 2014).

An important finding concerns the relationship between reversal in APOS theory and Polya's looking-back stage. S1 and S2 reported that they rechecked previously completed steps before proceeding to subsequent calculations, whereas comparable evidence was not identified for S3 and S4. However, none of the participants explicitly demonstrated a complete final verification by substituting the obtained values back into the original conditions. The procedural rechecking reported by S1 and S2 should therefore be distinguished from complete final verification. Polya's *looking back* involves reconsidering both the result and the reasoning used to obtain it and can extend beyond simply obtaining a correct answer. Similarly emphasizes that looking back provides opportunities to learn from the problem-solving experience rather than functioning merely as a final procedural requirement. Accordingly, the present findings indicate that correct numerical solutions did not necessarily correspond to fully documented reflective verification (Arnon et al., 2014; Stacey, 2022).

The limited final verification also resulted in restricted evidence of de-encapsulation. Although participants could operate on the mathematical Objects they

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had constructed, they did not systematically unpack the final solution and return to the underlying processes to verify it against the original conditions. For example, none explicitly demonstrated that 17,500 + 22,500 + 27,500 + 32,500 =100,000 while also verifying that each pair of adjacent shares differed by IDR 5,000. Within APOS theory, de-encapsulation involves returning from an Object to the Process from which it was constructed, whereas Polya's *looking back* encourages solvers to reconsider and examine their results and solution paths. The limited evidence observed in the present cases therefore suggests that obtaining an Object-level solution does not automatically ensure that the solver will unpack and verify that solution. This reinforces the educational importance of explicitly encouraging students to revisit both their results and the mathematical relationships underlying them (Arnon et al., 2014; Stacey, 2022)

Evidence of thematization was also not clearly identified in the available data. Although the participants developed Schemas appropriate to the given problem, the written solutions and interviews did not provide sufficient evidence that these Schemas were subsequently treated as mathematical entities and connected to broader conceptual structures beyond the immediate task. This absence of evidence should not be interpreted as indicating that the participants were incapable of thematization; rather, the data-collection task did not provide sufficient evidence to establish such a construction. APOS theory emphasizes that Schemas are dynamic structures that develop through the organization and interaction of Actions, Processes, Objects, and other Schemas. Consequently, claims about higher-level APOS constructions should be made only when supported by observable evidence from participants' mathematical activity and explanations (Arnon et al., 2014).

The gender comparison in the present study should be interpreted descriptively. In the four selected cases, the two male participants used Gauss–Jordan elimination and direct substitution, whereas the two female participants explicitly incorporated arithmetic-sequence relationships into their solution processes. However, all four participants demonstrated evidence of Action, Process, Object, and Schema structures, while the mathematical pathways through which these structures were constructed differed. A gender-focused APOS, involving one male and one female high-achieving junior-high-school student solving integer problems, similarly reported APOS-based descriptions of the thinking of both participants. Because that study involved a different

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mathematical topic, educational level, and only two participants, it should not be used to establish general gender differences; instead, it demonstrates the usefulness of APOS for examining how male and female participants may construct mathematical knowledge through individual pathways (Putri et al., 2025).

The differences observed between the male and female participants should therefore not be interpreted as evidence that gender determined strategy selection or mathematical ability. The four participants in the present study were purposively selected precisely because they obtained correct answers using distinctive strategies. Moreover, research on prospective teachers' metacognitive activity has shown that variations may occur both between and within gender groups. For example, examined awareness, regulation, and evaluation in mathematical problem solving and reported variations in metacognitive activities among male and female prospective teachers. Such findings support examining gender as one analytical lens while maintaining attention to individual variation. Thus, the present study characterizes the different APOS constructions observed in four selected cases rather than attributing particular mathematical strategies or mental structures inherently to male or female learners (Ramlah et al., 2024).

For mathematics teacher education, these findings highlight the value of providing opportunities for pre-service teachers to compare multiple representations and solution strategies. The four solutions in this study could be discussed simultaneously so that students examine how a system of linear equations, an augmented matrix, substitution, arithmetic-sequence relationships, and the arithmetic-series formula represent the same underlying contextual relationships. Multiple-representation-based and problem-based instruction in linear algebra can support dimensions of pre-service teachers' mathematical understanding. From an APOS perspective, comparing these representations may also provide opportunities for learners to coordinate different Processes and reorganize their existing mathematical Schemas (Arnon et al., 2014)

Finally, instruction should provide explicit opportunities for verification and reflection after a solution has been obtained. In the present study, all four participants reached the correct numerical result, but complete verification against the original problem conditions was not documented. Students could therefore be asked to substitute their final values into the original equations, verify the total amount, check the

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consecutive differences, compare alternative solution methods, and explain why the methods produce equivalent results. Emphasizes the continuing importance of Polya's *looking back* as a means of learning from the problem-solving experience, while research on classroom implementation of Polya's framework also highlights the role of planning and looking back in mathematical problem-solving instruction (Arnon et al., 2014). Such activities could simultaneously strengthen reflective verification within Polya's framework and provide opportunities for further coordination, de-encapsulation, and Schema development from an APOS perspective (Ambrus & Barczi-Veres, 2022; Arnon et al., 2014.; Stacey, 2022)

CONCLUSION

This study found similarities and differences in how two male and two female pre-service mathematics teachers solved a contextual system-of-linear-equations problem. All four successfully understood the problem, devised a plan, and carried out their strategies, obtaining IDR 17,500 for the youngest child and IDR 32,500 for the eldest. The male participants used Gauss–Jordan elimination (S1) and direct substitution (S2), while the female participants combined an arithmetic-sequence representation with substitution (S3) and used the arithmetic-series formula (S4). From the APOS perspective, all participants demonstrated Action, Process, Object, and Schema structures through interiorization, coordination, and encapsulation. Their mathematical connections and Schema organization differed: the male participants connected linear equations with matrix operations or substitution, whereas the female participants explicitly incorporated arithmetic-sequence or arithmetic-series concepts. Within the study's coding framework, reversal was identified in the male participants through their reported rechecking of preceding steps; comparable evidence was not identified in the female participants. However, none demonstrated complete final verification against the original conditions. Evidence of de-encapsulation associated with verification was therefore limited, and thematization was not clearly identified. These findings describe the four selected cases and do not establish that gender determines mathematical ability or APOS constructions. An unidentified mechanism also does not imply that a participant was incapable of it. Mathematics teacher education should encourage students to compare solution strategies, explain conceptual connections, and explicitly

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verify their answers. Further research with more participants and varied tasks is needed to examine these patterns across contexts.

DECLARATION:

DECLARATION OF SUBMISSION AND AUTHOR APPROVAL

Phenomenon: Jurnal Pendidikan MIPA has been selected as the target journal for this manuscript because the article examines pre-service mathematics teachers' mental constructions in solving systems of linear equations through APOS theory. This study is expected to contribute to the development of mathematics teaching and learning in higher education. All authors have read and approved the final manuscript and consented to its submission to this journal. The authors declare that the manuscript has not been published previously and is not currently under consideration for publication in any other journal.

Availability of Data and Materials: The research data, comprising participants' written work and interview transcripts, are not publicly available to protect participants' identities and confidentiality. Access to anonymized data may be considered upon reasonable request to the corresponding author, subject to participants' consent and applicable institutional requirements.

Competing Interests: The authors declare that they have no financial or non-financial competing interests related to this research or the publication of this article.

Funding: This research was funded by Universitas Kristen Satya Wacana.

Authors' Contributions: NR contributed to the formulation of the research problem, research design, instrument development, data collection and analysis, and preparation of the initial manuscript draft. SBW contributed to the development of the theoretical framework, methodological review, interpretation of the findings, and manuscript revision. WAR, KW, and SM contributed to reviewing the data analysis, discussing the findings, and editing the manuscript.

Informed Consent to Participate: All participants were informed of the purpose and procedures of the study and voluntarily provided informed consent to participate. Participants' identities were anonymized using codes, and their data were used in accordance with the consent provided.

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DECLARATION OF GENERATIVE AI USE

During the preparation of this manuscript, the authors used ChatGPT, developed by OpenAI, to assist with translation, grammatical revision, and improvements to writing clarity. All AI-generated output was reviewed, verified, and edited by the authors to ensure accuracy and consistency with the research data and objectives. The authors take full responsibility for the content and scientific integrity of the final manuscript.

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