ANALYSIS OF SPHERICAL ASTRONOMY ALGORITHMS FOR PREDICTING THE SOLAR ECLIPSE BY W.M. SMART

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Abstract

A solar eclipse is an unusual natural occurrence. Muslims used the phenomenon of a solar eclipse as the season for prayer, which was only carried out during a solar eclipse from early in the phenomenon until the end. Although there is no statistically significant difference in estimating a solar eclipse, as the seasons change, a solar eclipse becomes increasingly important. Lots of alternatives can be used as a reference in the determination of a solar eclipse. Among the alternatives commonly used is the calculation of spherical astronomy. Despite its complexity, many people still use it as the foundation for calculations based on a solar eclipse phytagoras as well as the determination of the celestial coordinate system.

Keywords: *solar eclipse, spherical astronomy, W.M. Smart*

Abstrak

Gerhana Matahari merupakan salah satu fenomena alam yang jarang sekali terlihat keberadaannya. Umat Islam menggunakan fenomena gerhana Matahari sebagai penentu waktu salat gerhana, sebab salat tersebut hanya dilaksanakan sepanjang berlangSungnya gerhana Matahari, yakni dari awal hingga berakhirnya fenomena gerhana Matahari. Meski tidak terdapat perbedaan yang signifikan perihal perkiraan gerhana Matahari sebagai penentu waktu salat gerhana, perhitungan gerhana Matahari memiliki peranan yang amat penting. Banyak alternatif perhitungan yang dapat digunakan sebagai acuan dalam penentuan perkiraan gerhana Matahari. Salah satu diantara alternatif perhitungan yang umum digunakan adalah, sistem perhitungan astronomi bola (*spherical astronomy*). Meski memiliki perhitungan yang cukup rumit, namun banyak kalangan yang masih menggunakannya sebagai dasar perhitungan data gerhana Matahari dengan berdasarkan pada perhitungan *Phytagoras* serta dari penentuan sistem koordinat langit.

Kata Kunci: *gerhana matahari, astronomi bola, W.M. Smart.*

A. Introduction

In the determination of the estimated time of the eclipse, there was no significant difference. In contrast to other calculations, the initial determination (Hijri calendar[1], namely ramadan and syawal especially), can be found to be the difference, both in terms of reference, method, and end result. Where all of these differences can lead to differences in Muslim worship. Either in *hisab*[2] or *rukyat*[3], not too many questioned the difference in estimating the eclipse. *Hisab* method uses the calculation to predict an eclipse, so *rukyat* uses its proceeds sourced from observation (either by using optical instruments or the naked eye) as a basis for estimating the eclipse. So therefore, there is no difference that is too conspicuous in the determination of a solar eclipse.

When estimating a solar eclipse, celestial sphere experts perform calculations that take into account all of the events that occur during a solar eclipse. Some events that count include when a solar eclipse occurs, the duration of the eclipse, the longitude and latitude where the eclipse occurred, the trajectory of a solar eclipse, the type of solar eclipse, and other events that occur only when a solar eclipse occurs. Various methods and a reference can be found on the celestial sphere. as a solar eclipse, which used data in an ephemeris book. [4] Data in an ephemeris can be obtained using several computer-calculated software programs.All data is based on Greenwich Mean Time[5], so all data is quite accurate if used in estimating the eclipse of the sun.

[1] Caliph Umar bin Khattab is thought to have established the Hijri adaptation calendar, which dates the process after the Prophet's journey to Medina.Caliph Umar bin Khattab believes that Muslims require their own calendar system to commemorate a few significant events in their religion. See Ibrahim Zein and Ahmed El Wakil, "On the Origins of the Hijri Calendar: A Multi-Faceted Perspective Based on the Covenants of the Prophet and Specific Date Verification", *Religions* 12 (2021):1, accessed 15 October, 2022, doi: 10.3390/rel12010042.

[2] *Hisab*, are calculation system or arithmetic. See Muhyiddin Khazin, *Kamus Ilmu Falak.....*, 30.

[3] *Rukyat,* are the activities observation or saw the celestial bodies. See Muhyiddin Khazin, *Kamus Ilmu Falak,* (Jogjakarta: Buana Pustaka, 2005), 69.

[4] *Ephemeris*, (Arabic called *Zaij*), is a table which mentions some data and exceeded all of celestial bodies. See Muhyiddin Khazin, *Kamus Ilmu Falak..........*, 92.

[5] Greenwich Mean Time (GMT) is time was measured from Greenwich mean mid-day, where the practice of calling mid-day 00h 00m GMT. See Howard Barnes, "A Briefer History of Time", *Georgi Dobrovolski Solar Observatory New Zealand,* (2010):1, accessed 16 October 2022.

Various algorithms Spherical astronomy has been used to predict a solar eclipse. One of them is spherical astronomy by W.M. Smart[6], in a book called Textbook On Spherical Astronomy[7]. Until now, W.M. Smart's spherical astronomy algorithms have been included in methodology calculations, but there are many others that are still used and have quite high accuracy.The algorithms of a textbook on spherical astronomy, also based on bessel elements. But, because the algorithms used in the textbook on spherical astronomy are for the calculation of spherical astronomy, the bessel elements that are used are bessel spherical functions.

B. Method

This research included in the qualitative study based on research literature[8]**.** This research attempts to develop the data and use it to deepen and expand existing knowledge. This research used astronomical and mathematical methods, where astronomical data that has been acquired will be reckoned mathematically. The primary data from the study is the work of W.M. Smart's astronomy book, Textbook on Spherical Astronomy. As for secondary data used, there are a few other astronomy books, spherical astronomy books, books that contain some data in an ephemeris book published by Ministry of Religious Affair, the book of spherical triangles, and all the other several references that deal with this research. Furthermore, the research is aided by the WinHisab application and a number of other official website pages that can present astronomical data in a formal and accurate manner.

[6] William Marshall Smart was an astronomer who was born in Pertshire. He has appointments as Chief Assistant and John Couch Adams Astronomer, Professor at Cambridge University, etc. He has many books about spherical astronomy, and one of them is A Textbook on Spherical Astronomy. See The Writer Team, *Former Fellows Of The Royal Society of Edinburgh 1783-2002*, (Edinburgh, The Royal Society of Edinburgh, 2006), accessed 16 October, 2022, https://web.archive.org/web/20061004113303/http://www.rse.org.uk/fellowship/fells_indexp2.pdf

[7] Spherical astronomy is the science of studying astronomical coordinate frames, directions, and apparent motions of celestial objects, etc. Concentrate on astronomical coordinates, apparent motion of stars, and time reckoning. See Markku Poutanen, et al., "Spherical Astronomy", *Fundamental Astronomy* (1994):9, accessed October 16th, 2022, doi: 10.1007/978-3-662-11794-1_2.

[8] The library research has an emic perspective, where the data in the research did not come from the perspective of private investigators but was based on some facts that were either conceptual or theoretical. See Amir Hamzah, *Metode Penelitian Kepustakaan: Library Research,* (Malang: Literasi Nusantara, 2020), 9.

C. Results and Discussion

C.1 The Algorithms of A Solar Eclipse by W.M. Smart at *Textbook on Spherical Astronomy*

a. The Overview of *Textbook on Spherical Astronomy* by W.M. Smart

Spherical astronomy algorithms are astronomy calculations that use astronomy positional data to determine the location of celestial objects. Elements that matter in spherical astronomy are a coordinate system and time. Coordinates of a celestial object based on the use of an equatorial coordinate system based on projections of the equatorial land and of the celestial sphere. The position of an object in this system is known in some terms as right ascension (α) and declination (δ). Longitude and time can be used to get where local object in a system of coordinates horizontal[9], which consists of *altitude*[10] and *azimuth*.[11]

Spherical astronomy specifically discusses various things about the direction of a celestial object. Include not only real motion (*diurnal motion*) problems, but also apparent motion (*annual motion*), where is much that affects true positions of celestial bodies on the celestial sphere or direction of celestial bodies from the center, as estimated by the observer.[12]

This textbook on spherical astronomy is based on lectures given at the University of Cambridge. Furthermore, the lecture was used in an astronomical observatory. This book contains some discussion of an important subject, such as heliografis coordinates, proper movement, determining the position on the sea's surface, the use of photography, proper astronomy in size, the orbit of a double star, and all matters concerning this problem.[13]

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[9] Coordinates to determine the position of the celestial bodies that based on the high and the azimuth of celestial bodies. See Muhyiddin Khazin, *Kamus Ilmu Falak*............., 46.

[10] The angular distance at celestial bodies, at above them and under horizon, measured along the great circle through noun and zenith point. See Jean Kovalevsky dan P.Kenneth Seidelmann, *Fundamentals of Astronomy*, (United Kingdom: University Press, Cambridge, 2004), 371.

[11] The angular distance measured (clockwise) along the horizon from a specified reference point (distances northward) until met with a great circle, which is evident from the point zenith which passed through an object on the celestial sphere. See Baca Jean Kovalevsky dan P.Kenneth Seidelmann, *Fundamentals of Astronomy*................., 372.

[12] William Chauvenet, *A Manual of Spherical and Practical Astronomy: Embracing The General Problems of Spherical Astronomy, The Special Applications to Nautical Astronomy, and The Theory and Use of Fixed and Portable Astronomical Instruments,* Vol.I., (Philadelphia: J.B. Lippincott Company, 1900), 18.

[13] William Marshall Smart, pengantar *Textbook on Spherical Astronomy* oleh William Marshall Smart, (Great Britain: University Press, Cambridge, 1977), v.

In practice, use almanacs from 1931 that adhere to the International Astronomical Union's recommendation.Even so, there are some modifications done by misconception to circumvent difficulties in understanding or material problems. Thus, a corner in the parallax (stellar parallax) stellar[14], Shown by symbols Π Than the π, where students (college students), has accustomed to use it (especially math where explained about a circle).[15]

b. Method of The Algorithms

a. Bessel Elements

Pict of 3.1

The scheme of the celestial sphere

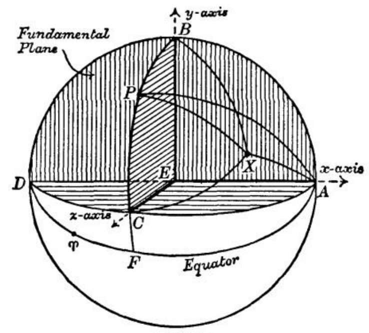

Fig. 144.

(Source: *Textbook on Spherical Astronomy*)

Methods used in the prediction the eclipse is in line with star ocultation[16] on eclipse. Across the center of the earth *E*, a line of *EC* described parallel to a straight line is connected to the Moon and Sun met with a circle, centered on a point *E*, in point C. *EC* is axis-z and plots is *DBA* (which shade), on *EC* which normal in point *E*, is

[14] The angle measured at an object observation with Astronomical Unit (AU). See Jean Kovalevsky and P.Kenneth Seidelmann, *Fundamentals of Astronomy*..............., 381.

[15] The difference symbol *Phi* between π with a ∏ is if symbol ∏ was the version uppercase (capital letters), so symbol π a version of a lowercase (small letters). As well as writing letters in Latin, the letters in the Greek language also comprises two kinds of letters (symbol), who referred to as ' upper' and ' lower'. Although essentially have the same purpose in symbol, phi if in the science of mathematics hence will have a distinct meaning.. If symbol of Π, have a function to *product,* as the outcome of a multiplication. Whereas symbol of π, has functions as a basic group , momentum conjugation , homotop group , major function of calculation and projection.

[16] Occultation is the nebulosity effect on one of the celestial bodies caused by the other celestial object having a greater diameter. Occultation truncates the source of main light on shadow objects, so this phenomenon is also known as an eclipse. See Jean Kovalevsky dan P.Kenneth Seidelmann, *Fundamentals of Astronomy*.............., 380.

fundamental areas. If P is north celestial pole, of past the point on the large C and P cut the base at EB lines. EA and EB is a x and y axis respectively.[17]

1) Elements of x, y and d. Put (α, δ) for right ascension and declination of Sun, and (α_1, δ_1) equal to Moon coordinate. (α, d) become right ascension and declination at C point in circle. (x, y, z) become Sun coordinate.[18]

$$x = r \cos \delta \sin (\alpha - a)$$

$$y = r [\sin \delta \cos d - \cos \delta \sin d \cos (\alpha - a)]$$

$$z = r [\sin \delta \sin d + \cos \delta \cos d \cos (\alpha - a)]$$

In a same way, found relations number for Moon:

$$x_1 = r_1 \cos \delta_1 \sin (\alpha_1 - a),$$

$$y_1 = r_1[\sin \delta_1 \cos d - \cos \delta_1 \sin d \cos(\alpha_1 - a)]$$

$$z_1 = r_1[\sin \delta_1 \sin d + \cos \delta_1 \cos d \cos(\alpha_1 - a)]$$

$$a = \alpha - \frac{b \sec \delta \cos \delta_1}{1-b} (\alpha_1 - a)$$ [19]

$$d = \delta - \frac{b}{1-b} (\delta_1 - \delta)$$

The calculations of a and d is the space of an hour.[20]

2) Elements of μ. Symbol μ showing the angle time at the C to the meridian in an ephemeris when time sideris in an ephemeris is G, then μ = G – α.[21]

3) Elements of f_1 and f_2

$$sinf_1 = \frac{R+k}{r\,(1-b)}$$

$$sinf_2 = \frac{R-k}{r\,(1-b)}$$ [22]

4) Elements of L_1 and L_2.

$$l_1 = z_1 \tan f_1 + k\, secf_1$$

$$l_2 = z_1 \tan f_2 - k\, secf_2$$

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[17] William Marshall Smart, *Textbook on Spherical Astronomy*, (Great Britain: University Press, Cambridge, 1977), 390-391.

[18] William Marshall Smart, *Textbook on Spherical Astronomy*.................., 391.

[19] Where $b = \frac{\sin P}{\sin P_1}$. See William Marshall Smart, *Textbook on Spherical Astronomy* ...., 392.

[20] William Marshall Smart, *Textbook on Spherical Astronomy*, ..., 391-392.

[21] William Marshall Smart, *Textbook on Spherical Astronomy*, (Great Britain: University Press, Cambridge, 1977), 392.

[22] William Marshall Smart, *Textbook on Spherical Astronomy*................, 393.

Value of x, y, $\sin d$, $\cos d$, μ, $l_1$and l_2 called bessel spherical function in an eclipse. This is a matter of first values at a distance every hour and lapped in 10 minutes in a further in an astronomical ephemeris. One point also tend to on the value $\tan f_1$, $\tan f_2$, μ` dan $d$`, so long as those values are constant, with needed precision in it [23]

c. **The Calculation of A Solar Eclipse (March 9th, 2016) in *Textbook on Spherical Astronomy* W.M. Smart.**

1. Calculate Bessel spherical functions (x, y, $\sin d$, $\cos d$, μ, l_1 and l_2)

a) Elements of x, y, $\sin d$ and $\cos d$.

Data from March 9th, 2016

Sun[24]		Moon[25]	
α	349° 44' 53,38"	α_1	0° 22' 25,75"
δ	-4° 24' 41,1"	δ_1[26]	347° 44' 31,52"
r	1° 00' 36"	r_1	0° 00' 09,36"

B	a	d
0° 00' 09,27"	350° 37' 55,84"	-5° 19' 13,99"
	a`	d`
	350° 40' 00,23"	-5° 18′ 54,87"

Descriptions:

α = right ascension of Sun α_1 = coordinates of Moon (*lattitude*)

[23] William Marshall Smart, *Textbook on Spherical Astronomy*, (Great Britain: University Press, Cambridge, 1977), 393-394.

[24] Sun database which contained right ascension (RA), declination and geocentric distance, from Epemeris database on March 9th, 2016.

[25] Moon database, Moon coordinates (*lattitude* dan *longitude*). See at WinHisab 2010 application, Epemeris database March 9th, 2016.

[26] NASA, "Moon Fact", diakses 26 November 2018, https://nssdc.gsfc.nasa.gov/planetary/factsheet/Moonfact.html.

$\boldsymbol{\delta}$ = declination of Sun $\boldsymbol{\delta_1}$ = coordinates of Moon (*longitude*)

$\boldsymbol{r}$ = distance geocentric of Sun (1 AU) $\boldsymbol{r_1}$ = distance Moon from Earth (AU)

$\boldsymbol{b}$ = $\frac{r_1}{r}$ [27] $\boldsymbol{a}$ = $\alpha - \frac{b \sec\delta \cos\delta_1}{1-b}(\alpha_1 - a)$[28]

$\boldsymbol{d} = \delta - \frac{b}{1-b}(\delta_1 - \delta)$[29]

Then,

x = $r \cos\delta \sin(\alpha - a)$...(5)[30] = 1° 00' 36" ×

cos(−4° 24' 41,1")×sin(349° 44' 53,38" − 350° 37' 55,84")

$\boldsymbol{x}$ = **-0° 0' 55,93"**

Whereas:

$x`$ = $r \cos\delta` \sin(\alpha` - a`)$

= 1° 00' 36" × cos(−4° 24'19,96")×sin(349° 45' 43, 22" −

350° 40' 0,23")

= 1° 00' 36" × 0° 59' 49,36" × (-0° 0' 56,84")

$\boldsymbol{x`}$ = **-0° 0' 57,24"**

As for:

y = $r\,[\sin\delta \cos d - \cos\delta \sin d \cos(\alpha - a)]$.....(6)[31]

= 1° 00' 36" × [sin(−4° 24'41,1") × cos −5° 19'13,99" -

cos(−4° 24' 41,1") × sin(−5° 19'13,99") × cos(349° 44'53,38" −

350° 37' 55,84")]

$\boldsymbol{y}$ = **0° 0' 57,65"**

Whereas:

$y`$ = $r\,[\sin\delta` \cos d` - \cos\delta` \sin d` \cos(\alpha` - a`)]$

= 1° 00' 36" × [sin(−4° 24'19,96") × cos(−5° 18'54,87") -

cos(−4° 24'19,96") × sin(−5° 18'54,87") × cos(349° 45' 43,22" −

350° 40' 0,23")]

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[27] William Marshall Smart, *Textbook on Spherical Astronomy*, (Great Britain: University Press, Cambridge, 1977), 392.

[28] William Marshall Smart, *Textbook on Spherical Astronomy*.........., 392.

[29] William Marshall Smart, *Textbook on Spherical Astronomy*.........., 392.

[30] William Marshall Smart, *Textbook on Spherical Astronomy*.........., 391.

[31] William Marshall Smart, *Textbook on Spherical Astronomy*.........., 391.

= 1° 00’ 36” × [(-0° 04’ 36,54” × 0° 59’ 44,52”) – (0° 59’ 49,36”× (-0° 05’ 33,49”) × 0° 59′ 59,55")]

= 1° 00’ 36” × [-0° 04’ 35,33” – (-0° 05’ 32,46”)]

y` **= 0° 0’ 57,7”**

sin *d* = sin(−5° 19’ 13,99”)

= **-0° 05′33,82"**

cos *d* = cos(−5° 19’ 13,99”)

= **0° 59’ 44,49”**

Therefore, the values of *x, y* sin *d* dan cos *d* are:

X	Y	**sin *d***	**cos *d***
-0° 0’ 55,93”	**0° 0’ 57,65”**	**-0°** 05’ 33,82”	**0° 59’ 44,49”**

b) Element of μ.

G	A
167° 05’ 22,53”	350° 37’ 55,84”
G`	***a*`**
182° 07’ 50,08”	350° 40’ 0,23”

Descriptions:

G = Sidereal epimeris time[32].

Then,

μ = G - α[33]

= 167° 05’ 22,53” - 350° 37’ 55,84”

$\boldsymbol{\mu}$ **= -183° 32’ 33,31”**

Variation of μ in each time (μ`):

μ` = G` - α`

μ` = 182° 07’ 50,08” – 350° 40’ 0,23”

= -168° 32’ 10,15”

[32] NASA, “Earth Fact”, diakses 26 November 2018, https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html.

[33] William Marshall Smart, *Textbook on Spherical Astronomy*, (Great Britain: University Press, Cambridge, 1977), 392.

c) Elements of f_1 and f_2.

R	K	r	B
0° 16' 6,48"	0° 16' 33,02"	1° 00' 36"	0° 0' 9,27"

Descriptions:

R = The distance semidiameter of the Sun.[34]

k = The distance semidiameter of the Moon.

$sinf_1 = \frac{R+k}{r\,(1-b)}$.[35]

$= \frac{0^\circ\, 16'6{,}48'' + 0^\circ\, 16'\, 33{,}02''}{1^\circ\, 00'\, 36'' \times (1-0^\circ\, 0'\, 9{,}27'')}$

$sinf_1$ = 0° 32' 25,11"

$\boldsymbol{f_1}$ = **32° 42' 16,67"**

$sinf_2 = \frac{R-k}{r\,(1-b)}$.[36]

$= \frac{0^\circ\, 16'6{,}48'' + 0^\circ\, 16'\, 33{,}02''}{1^\circ\, 00'\, 36'' \times (1-0^\circ\, 0'\, 9{,}27'')}$

$sinf_2$ = -0° 0' 26,35"

$\boldsymbol{f_2}$ = **-0° 25' 9,76"**

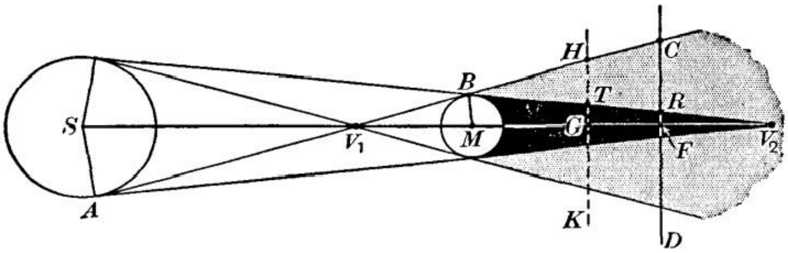
d) Elements of L_1 and L_2.

$l_1 = z_1 \tan f_1 + k\, secf_1$.

$l_2 = z_2 \tan f_2 + k\, secf_2$.

Pict of 3.2

The scheme of trigonometry diagram on lunar eclipse



(Source: *Textbook on Spherical Astronomy*)

[34] WinHisab 2010, Epemeris on March 9th, 2016.
[35] William Marshall Smart, *Textbook on Spherical Astronomy*............., 393.
[36] William Marshall Smart, *Textbook on Spherical Astronomy*..........., 393.

Side of MF (z coordinate):

$\boldsymbol{V_1}$F = $\boldsymbol{V_1}$M + $\boldsymbol{MF}$

$0^\circ\ 25'05{,}19''$ = $k \csc f_1$ + MF

$0^\circ\ 25'05{,}19''$ = (0° 16' 33,02" × (csc(32° 42' 16,67"))) + MF

$0^\circ\ 25'05{,}19''$ = $0^\circ\ 30'37{,}88''$ + MF

$0^\circ\ 25'05{,}19''$ - $0^\circ\ 30'37{,}88''$ = MF

- $\mathbf{0^\circ\ 05'32{,}69''}$ = *MF*.

From the explanation above, it has already been known that the value of a point z coordinate is: - 0° 5' 32,69". But, the coordinate of z (MF), measured in $\overrightarrow{FM}$ function, which means that a line (the coordinates) having value by a positive direction. So, the result is:

$z(\boldsymbol{z_1})$ **= $\mathbf{0^\circ}$ 5' 32,69".**

Looking for value l_1 and l_2 (as the circle radius which conical penumbra and umbra intersecting in the basic plane):

l_1 = z_1 tan f_1 + $k\ sec f_1$[37]

= (0° 5' 32,69" × tan 32° 42' 16,67") + (0° 16' 33,02" × (sec(32° 42' 16,67")))

= 0° 3' 33,62" + 0° 19' 40,11"

$\boldsymbol{l_1}$ **= $\mathbf{0^\circ}$ 23' 13,73"**

l_2 = z_1 tan f_2 - $k\ sec f_2$.[38]

= (0° 5' 32,69" × tan(-0° 25' 9,76")) - (0° 16' 33,02" × (sec((-0° 25' 9,76")))

= -0° 0' 2,44" - 0° 16' 33,05"

$\boldsymbol{l_2}$ **= - $\mathbf{0^\circ}$ 16' 35,49"**

2. The Algorithms of Solar Eclipse

In algorithms a solar eclipse , first we count radius on the *KH* plane which called z = ζ, determined by L_1 and L_2. Because the values of *KH* plane is unknown, then we should search the values first:

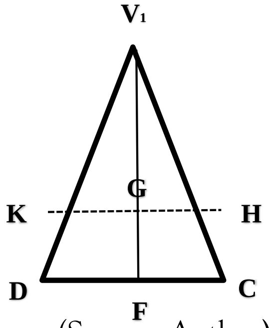
[37] William Marshall Smart, *Textbook on Spherical Astronomy*.............., 394.
[38] William Marshall Smart, *Textbook on Spherical Astronomy*.............., 393.

KH = *CD*

Use a triangle of DV_1C,

Pict of 3.3

The scheme of triangle DV_1C,



(Source: Author)

V_1C = $0^\circ\ 29'48{,}76"$.

V_1F = $0^\circ\ 25'05{,}19"$.

Then, the side of *FC* are:

FC = $\sqrt{V_1C^2 - V_1F^2}$

= $\sqrt{(0^\circ\ 29'48{,}76")^2 - (0^\circ\ 25'05{,}19")^2}$

= $\sqrt{0^\circ\ 14'48{,}8" - 0^\circ\ 10'\ 29{,}33"}$

= $\sqrt{0^\circ\ 04'19{,}47"}$

FC = $\mathbf{0^\circ\ 16'\ 6,48"}$

Because *FC*=*DF*, then:

CD = $(\mathbf{0^\circ\ 16'\ 6,48"})^2$

= $\mathbf{0^\circ\ 04'19,47"}$

With *KH* as large as side *CD* , because *KH* line with the *CD* are align.

KH (**ζ**) = $\mathbf{0^\circ\ 04'19,47"}$**.**

At the picture, line of G*H* = L_1 and GT = L_2, and in example G*H* line and G*T*, line of *F*G = ζ. Then, *F*G = *KH*. Therefore calculate L_1 and L_2 with formulae:

L_1 = l_1 – ζ tan f_1 ....(21).[39]

= 0° 23' 13,73" – (0° 04'19,47" × (tan 32° 42' 16,67"))

= 0° 23' 13,73" - 0° 02' 46,61"

[39] William Marshall Smart, *Textbook on Spherical Astronomy*..........., 393.

$\boldsymbol{L_1}$ = **$\mathbf{0}^\circ$ 20' 27,12".**[40]

L_2 = l_2 – $\zeta \tan f_2$ ....(22).[41]

= (- 0° 16' 35,49") – ($0^\circ\ 04'19{,}47''$ × ($\tan -0^\circ$ 25' 9,76"))

= (- 0° 16' 35,49") – (-0° 00' 01,9")

$\boldsymbol{L_2}$ = **- $\mathbf{0}^\circ$ 16' 33,59".**[42]

Calculate triangle of *APX* or (ξ, η, ζ), use formulae:

ξ = ρ cos Φ\` $\sin h$.

η = ρ [$\sin \Phi\text{\`} \cos d - \cos \Phi\text{\`} \sin d \cos h$],

ζ = ρ [$\sin \Phi\text{\`} \sin d + \cos \Phi\text{\`} \cos d \cos h$].

Where:

Φ\` **= geocentric latitude**

= $\tan \Phi$: 1 - $e^2 \times \frac{R_N}{R_N+h} \times \tan \Phi$

h (XPC) = μ – λ – 1.0027Δ*T*.[43]

= (-183° 32' 33,31") - 0° 0' 5,31"- 1,0027 × 69

Which λ is west longitude of *Greenwich*.

XPC **= -252° 43' 49,3".**

Next calculations is search a value of geocentric langitude (Φ\`).

Φ\` = $\tan \Phi$\` : 1 - $e^2 \times \frac{R_N}{R_N+h} \times \tan \Phi$.[44]

where[45],

e[46] = $\sqrt{2f - f^2}$ (which *f* is formulae parameter of *ellipsloid flattening*).[47]

= $\sqrt{(2 \times 0^\circ\ 0'12{,}07'') - (0^\circ\ 0'12{,}07'')^2}$

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[40] L_1 always positive.

[41] William Marshall Smart, *Textbook on Spherical Astronomy*.........., 394.

[42] L_2 is negative.

[43] William Marshall Smart, *Textbook on Spherical Astronomy*............, 395.

[44] James R. Clynch, "Geodetic Coordinate Conversions", *Naval Postgraduate* (2002): 1, accessed August 20th, 2018.

[45] George H. Born, "Geodetic and Geocentric Latitude", (tt: tp, tth): 2, accessed August 20th, 2018.

[46] *e,* is value of eccentricity, results from formulae quadrant multiplication roots *f* or *flattening* (*ellipticity*), for WGS-84. See George H. Born, *Geodetic and Geocentric Latitude*, (tt: tp, tth), 2, accessed August 20th, 2018.

[47] Formulae Parameter of *Ellipsoid Flattening* (*f*) for *World Geodetic System* 84 (WGS 84) is (1/298.257223563). See B. Louis Decker, "World Geodetic System 1984", *Defence Mapping Agency* (1986): 22, accessed August 20th, 2018.

$= \sqrt{0^\circ\ 0'\ 24{,}14'' - \ 0^\circ\ 0'\ 00{,}04''}$

$= 0^\circ\ 04'\ 54{,}75''$

$R_N \quad = \frac{a^{48}}{\sqrt{1-e^2 sin^2 \Phi}}.$[49]

$= \frac{6378{,}137}{\sqrt{1-(0^\circ\ 04'\ 54{,}75'')^2 sin^2(-02^\circ 46' 20'')}}$

$= \frac{6378{,}137}{\sqrt{1-0^\circ\ 00' 24{,}13'' \times 0^\circ\ 0'\ 8{,}42''}}$

$= \frac{6378{,}137}{0^\circ\ 59' 59{,}97''}$

= 6378,190152.

Then, geocentric latitude is:

$\Phi` \quad = 1 - e^2 \times \frac{R_N}{R_N + h} \times \tan \Phi.$

$= 1 - (0^\circ\ 04'\ 54{,}75'')^2 \times \frac{6378{,}190152}{6378{,}190152+(-252^\circ\ 43'\ 49{,}3'')} \times \tan(-2^\circ\ 46'\ 20'')$

$= 1 - 0^\circ\ 00'\ 24{,}13'' \times 1^\circ\ 02'\ 28{,}53'' \times (-0^\circ\ 02'\ 54{,}32'')$

$\tan \Phi` \quad = 1^\circ\ 00'\ 01{,}22''$

$\Phi` \quad = 45^\circ\ 0'\ 34{,}94''.$

Calculate the ρ (distance observer):

$r \quad = \sqrt{x^2 + y^2 + z^2}.$[50]

Before calculate the value of *r*, must calculate this value first (*x*, *y*, *z*)[51]:

$x \quad = (R_N + h) \times \cos \Phi \times \cos \lambda.$

$y \quad = (R_N + h) \times \cos \Phi \times \sin \lambda.$

$z \quad = ([1 - e^2] \times R_N + h) \times \sin \Phi.$[52]

then,

$x \quad = (R_N + h) \times \cos \Phi \times \cos \lambda.$

$= (6378{,}190152 + (-252^\circ\ 43'\ 49{,}3'')) \times \cos(-2^\circ\ 46' 20'') \times \cos(104^\circ\ 24' 53'')$

= -1523,081909.

[48] *a* is radius at equator.

[49] James R. Clynch, *Geodetic Coordinate Conversions*, 1.

[50] James R. Clynch, *Geodetic Coordinate Concertions,* 4.

[51] *x*, *y*, dan *z* are Cartesian coordinates.

[52] James R. Clynch, *Geodetical Coordinate Conversions*, 3.

y $= (R_N + h) \times \cos \Phi \times \sin \lambda.$

$= (6378{,}190152 + (-252^\circ\ 43'\ 49{,}3")) \times \cos(-2^\circ\ 46'20") \times \sin(104^\circ\ 24'53")$

= 4845,703746.

z $= ([1 - e^2] \times R_N + h) \times \sin \Phi.$

$= ([1 - (0^\circ\ 04'\ 54{,}75")^2\ ] \times 6378{,}190152 + (-252^\circ\ 43'\ 49{,}3")) \times \sin(-2^\circ\ 46'20").$

= 77442,65873.

After that, calculate the value of *r*:

r $= \sqrt{x^2 + y^2 + z^2}.$

$= \sqrt{(-1523{,}081909)^2 + (4845{,}703746)^2 + (77442{,}65873)^2}$

***r* (ρ) = 6020847759.**

After calculate triangle of *APX*:

ξ $= \rho \cos \Phi`\ \sin h.$[53]

$= 6020847759 \times \cos(45^\circ\ 0'\ 34{,}94") \times \sin(-252^\circ\ 43'\ 49{,}3")$

= 4064763292.

η $= \rho\ [\sin \Phi` \cos d - \cos \Phi` \sin d \cos h].$[54]

$= 6020847759 \times [\ \sin(45^\circ\ 0'\ 34{,}94") \times 0^\circ\ 59'\ 44{,}49" - \cos(45^\circ\ 0'\ 34{,}94") \times -0^\circ 5'\ 33{,}82" \times \cos(-252^\circ\ 43'\ 49{,}3")]$

= 4122588014.

ζ $= \rho\ [\sin \Phi` \sin d + \cos \Phi` \cos d \cos h].$[55]

$= 6020847759 \times [(\ \sin(45^\circ\ 0'\ 34{,}94") \times (-0^\circ 5'\ 33{,}82")) + (\cos(45^\circ\ 0'\ 34{,}94") \times 0^\circ\ 59'\ 44{,}49" \times \cos(-252^\circ\ 43'\ 49{,}3"))]$

= -1653071860.

Calculate a correction of each hour (ξ`, η`, ζ`):

ξ` $= \mu`\rho \cos \Phi`\ \sin h$

η` $= \mu`\rho\ [\sin \Phi` \cos d - \cos \Phi` \sin d \cos h]$

ζ` $= \mu`\rho\ [\sin \Phi` \sin d + \cos \Phi` \cos d \cos h]$

[53] William Marshall Smart, *Textbook on Spherical Astronomy*, (Great Britain: University Press, Cambridge, 1977), 395.
[54] William Marshall Smart, *Textbook on Spherical Astronomy*..........., 395.
[55] William Marshall Smart, *Textbook on Spherical Astronomy*..........., 395.

So, the calculations are:

ξ` = μ`ρ cos Φ` sin h.[56]

= (- 168° 32' 10,15") × 6020847759 × cos(45° 0' 34,94") × sin(−252° 43' 49,3")

= 2,12974253 × 10^{11}.

η` = μ`ρ [$\sin \Phi`\cos d - \cos \Phi`\sin d\cos h$].

= (- 168° 32' 10,15")×6020847759×[(sin(45° 0' 34,94") × 0° 59' 44,49") − (cos(45° 0' 34,94") × (−0°5' 33,82") × cos(−252° 43' 49,3"))]

= -6,948057164 × 10^{11}.

ζ ` = μ`ρ [$\sin \Phi`\sin d + \cos \Phi`\cos d\cos h$].

= (-168° 32' 10,15")×6020847759× [(sin(45° 0' 34,94") × (−0°5'33,82")) + cos(45° 0' 34,94") × 0° 59' 44,49" × cos(−252° 43' 49,3")]

= 2,786027197 × 10^{11}.

Calculate the value of L_2, with the formulae quadratic equations:

$(x-\xi)^2 + (y-\eta)^2 = {L_2}^2$.

Where,

(*x*, *y*) = Cartesian coordinate at the center of circle.

(ξ,η) = coordinate the observer.

Therefore, a calculation are:

$((-1523{,}081909) - 4064763292)^2 + (4845{,}703746 - 4122588014)^2 = {L_2}^2$.

$5789473636 = L_2$.

Because *T* Is a time that has calculated in L_2, then:

T = -0° 16′ 33,59".

T + *t*, become an Ephemeris time According to the beginning (or end) during the eclipse, then:

T + *t* = 23° 30′59" (the estimate of early eclipse).

So, it is estimated *t* value is:

(-0° 16′ 33,59") + *t* = 23° 30′59"

t = 23°30′ 59" – (-0° 16′ 33,59")

[56] William Marshall Smart, *Textbook on Spherical Astronomy*..........., 395.

= 23° 47’ 32,59”.

At $T + t$ time, values of t expressed in a unit of hours, then:

x $= x_o + x`t$

x_o $= x`t - x$

= ((-0° 0’ 57,24”) × 23° 47’ 32,59”) – (-1523,081909)

= 1522,70361

y $= y_o + y`t,$

y_o $= y`t - y$

= (0° 0’ 57,65” × 23° 47’ 32,59”) – (4845,703746)

= -4845,322737.

ξ $= \xi_o + \xi`t,$

ξ_o $= \xi`t - \xi$

= ((2,12974253 × 10^{11})× 23° 47’ 32,59”) – (4064763292)

= 5,063100896 × 10^{12}.

η $= \eta_o + \eta`t.$

η_o $= \eta`t - \eta$

= ((-6,948057164 × 10^{11}) × 23° 47’ 32,59”) – (4122588014)

= -1,653520846 × 10^{13}.

Each of (x, y, hingga η_o) values have been known, hence to preliminary calculations or the end of ring phase of eclipse can be taken into calculation that using formulaes:

$$[x_o - \xi_o + t\,(x` - \xi`)]^2 + [y_o - \eta_o + t\,(y` - \eta`)]^2 = {L_2}^2$$

$$[(1522{,}70361 - (5{,}063100896 \times 10^{12})) + (23^\circ\,47'\,32{,}59'' \times ((-0^\circ\,0'\,57{,}24'') - (-2{,}12974253 \times 10^{11})))]^2 + [((-4845{,}322737) - (-1{,}653520846 \times 10^{13})) + (23^\circ\,47'\,32{,}59'' \times (0^\circ\,0'\,57{,}65'' - (-6{,}948057164 \times 10^{11})))]^2 = {L_2}^2$$

$$3{,}306629459 \times 10^{13} = L_2.$$

Then counting the fund, (M and m), (N and n).

$m \sin M = x_o - \xi_o,$ $\quad m \cos M = y_o - \eta_o,$

$n \sin N = x` - \xi`$ $\quad n \cos N = y` - \eta`.$

Known that, **values of M obtained through tan M = (x_o - ξ_o)/(y_o - η_o)**, that will bring two values at M:

tan M = (x_o - ξ_o)

= $(1522{,}70361 - (5{,}063100896 \times 10^{12}))$

= $-5{,}063100894 \times 10^{12}$

M = -90

or,

tan M = (y_o - η_o)

= $((-4845{,}322737) - (-1{,}653520846 \times 10^{13}))$

= $1{,}653520846 \times 10^{13}$

M = 90

Obtained by formulae and the m:

$\sqrt{[(x_o - \xi_o)^2 + (y_o - \eta_o)^2]} = m$

So , the calculation can be described as:

$\sqrt{[(x_o - \xi_o)^2 + (y_o - \eta_o)^2]} \times \sin M = x_o - \xi_o,$

$\sqrt{\left[\left(1522{,}70361 - (5{,}063100896 \times 10^{12})\right)^2 + \left((-4845{,}322737) - (-1{,}653520846 \times 10^{13})\right)^2\right]}$

$\times \sin 90^\circ = -5{,}063100894 \times 10^{12}$

= $(1{,}729300753 \times 10^{13}) \times 1 = -5{,}063100894 \times 10^{12}$.

= $1{,}729300753 \times 10^{13} = -5{,}063100894 \times 10^{12}$

or,

$\sqrt{\left[\left(1522{,}70361 - (5{,}063100896 \times 10^{12})\right)^2 + \left((-4845{,}322737) - (-1{,}653520846 \times 10^{13})\right)^2\right]}$

$\times \sin -90^\circ = -5{,}063100894 \times 10^{12}$

= $(1{,}729300753 \times 10^{13}) \times -1 = -5{,}063100894 \times 10^{12}$.

= $-1{,}729300753 \times 10^{13} = -5{,}063100894 \times 10^{12}$

and

$\sqrt{[(x_o - \xi_o)^2 + (y_o - \eta_o)^2]} \times \cos M = y_o - \eta_o.$

$\sqrt{\left[\left(1522{,}70361 - (5{,}063100896 \times 10^{12})\right)^2 + \left((-4845{,}322737) - (-1{,}653520846 \times 10^{13})\right)^2\right]}$

$\times \cos 90^\circ = -5{,}063100894 \times 10^{12}$

= $(1{,}729300753 \times 10^{13}) \times 0$ = -$5{,}063100894 \times 10^{12}$.

= 0 = -$5{,}063100894 \times 10^{12}$

or,

$\sqrt{\left[\left(1522{,}70361 - (5{,}063100896 \times 10^{12})\right)^2 + \left((-4845{,}322737) - (-1{,}653520846 \times 10^{13})\right)^2\right]}$

$\times \cos -90^{\circ}$ = -$5{,}063100894 \times 10^{12}$

= $(1{,}729300753 \times 10^{13}) \times 0$ = -$5{,}063100894 \times 10^{12}$.

= 0 = -$5{,}063100894 \times 10^{12}$

For the **value of *N* obtained through tan *N* =** $(\boldsymbol{x`} - \boldsymbol{\xi`})/(\boldsymbol{y`} - \boldsymbol{\eta`})$, that will enable two values in *N*:

tan *N* = $(x` - \xi`)$

= $((-0^{\circ}\ 0'\ 57{,}24'') - 2{,}12974253 \times 10^{11})$

= -$2{,}12974253 \times 10^{11}$

N = -90

or,

tan *N* = $(y` - \eta`)$

= $(0^{\circ}\ 0'\ 57{,}65'' - (-6{,}948057164 \times 10^{11}))$

= $6{,}948057064 \times 10^{11}$

N = 90

One of *n* obtained through formulae:

$\sqrt{[(x` - \xi`)^2 + (y` - \eta`)^2]} = n$

So , the calculation can be described as:

$\sqrt{[(x` - \xi`)^2 + (y` - \eta`)^2]} \times \sin N = x` - \xi`$,

$\sqrt{[((-0^{\circ}\ 0'\ 57{,}24'') - 2{,}12974253 \times 10^{11})^2 + (0^{\circ}\ 0'\ 57{,}65'' - (-6{,}948057164 \times 10^{11}))^2]}$

$\times \sin 90^{\circ}$ = -$2{,}12974253 \times 10^{11}$.

= $(7{,}267138378 \times 10^{11}) \times 1$ = -$2{,}12974253 \times 10^{11}$.

= $7{,}267138378 \times 10^{11}$ = -$2{,}12974253 \times 10^{11}$.

or,

$\sqrt{[((-0^{\circ}\ 0'\ 57{,}24'') - 2{,}12974253 \times 10^{11})^2 + (0^{\circ}\ 0'\ 57{,}65'' - (-6{,}948057164 \times 10^{11}))^2]}$

$\times \sin -90^{\circ}$ = -$2{,}12974253 \times 10^{11}$.

= $(7{,}267138378 \times 10^{11})$ × -1 = -$2{,}12974253 \times 10^{11}$.

= -$7{,}267138378 \times 10^{11}$ = -$2{,}12974253 \times 10^{11}$,

and

$\sqrt{[(x` - \xi`)^2 + (y` - \eta`)^2]}$ × $\cos N$ = $y` - \eta`$.

$\sqrt{[((-0^\circ\ 0'\ 57{,}24'') - 2{,}12974253 \times 10^{11})^2 + (0^\circ\ 0'\ 57{,}65'' - (-6{,}948057164 \times 10^{11}))^2]}$
× $\cos 90^\circ$ = $6{,}948057064 \times 10^{11}$

= $7{,}267138378 \times 10^{11}$ × 0 = $6{,}948057064 \times 10^{11}$.

= 0 = $6{,}948057064 \times 10^{11}$.

or,

$\sqrt{[((-0^\circ\ 0'\ 57{,}24'') - 2{,}12974253 \times 10^{11})^2 + (0^\circ\ 0'\ 57{,}65'' - (-6{,}948057164 \times 10^{11}))^2]}$
× $\cos -90^\circ$ = $6{,}948057064 \times 10^{11}$

= $7{,}267138378 \times 10^{11}$ × 0 = $6{,}948057064 \times 10^{11}$.

= 0 = $6{,}948057064 \times 10^{11}$.

That need to be considered in counting it is , that results in determining a values of *M* and *N,* the values was chosen for $\sin M$ or $\sin N$, must have an equality of values (positve or negative) with (x_o - ξ_o).[57]

At formulaee m sin M = x_o - ξ_o, the values of -90 is the exact value to apply to the number of M. This is because , if we use the -90 on a calculation, the number of $\sin -90^\circ$ the value will be the same as value (x_o - ξ_o), have negative values[58].

And the formulaee of n sin N = $x`$ - $\xi`$, the value of -90 is the exact value to apply to the number of N. This is because , if we use the -90 on a calculation, the number of $\sin -90^\circ$ the value will be the same as value (x_o - ξ_o), have negative values[59].

After the value of N, n, M, and m has known, then:

$$n^2t^2 + 2mnt\ \cos\ (M - N) + m^2 - {L_2}^2 = 0.$$

Therefore,

[57] William Marshall Smart, *Textbook on Spherical Astronomy*............., 396.
[58] The value of $\sin -90^\circ$ = -1, and the value of (x_o- ξ_o) is -9855404725.
[59] The value of $\sin -90^\circ$ = -1, and the value of ($x`$- $\xi`$) is -638374568,8.

$((7{,}267138378 \times 10^{11})^2 \times (23^\circ 47' 32{,}59'')^2) + ((2 \times (1{,}729300753 \times 10^{13}) \times (7{,}267138378 \times 10^{11}) \times 23^\circ 47' 32{,}59'') \times \cos((-90) - (-90))) + ((1{,}729300753 \times 10^{13})^2) - (5789473636)^2 = 0$.

$20282417{,}31 + (5{,}980010621 \times 10^{26}) + 4157790{,}045 = 0$.

Then calculate the angle of Ψ, with formulae:

$L_2 \sin \Psi = m \sin (M - N)$

$L_2 \sin \Psi = m \sin (M) - m \sin (N)$

$\sin \Psi = \frac{m \sin (M) - m \sin (N)}{L_2}$

$\sin \Psi = \frac{((1{,}729300753 \times 10^{13}) \times \sin(-90^\circ)) - ((1{,}729300753 \times 10^{13}) \times \sin(-90^\circ))}{5789473636}$

Ψ = $-\mathbf{2986{,}974053}$ dan $+\mathbf{2986{,}974053}$.

After found the value of Ψ, then calculate value of t, with calculate the value of $\frac{L_2 \cos \Psi}{n}$, first:

$\frac{L_2 \cos \Psi}{n} = \frac{(3{,}306629459 \times 10^{13})(\cos(2986{,}974053)^{60})}{(7{,}267138378 \times 10^{11})}$

$= -\mathbf{13^\circ\ 17'0{,}72''}$ **(for the early phase of eclipse)**

and

$\frac{L_2 \cos \Psi}{n} = \frac{(3{,}306629459 \times 10^{13})(\cos(-2986{,}974053)^{61})}{(7{,}267138378 \times 10^{11})}$

$= \mathbf{13^\circ\ 17'0{,}72''}$ **(for the end phase of eclipse)**

After that, calculate:

$t_1 = -\frac{m}{n} \cos(M - N) + \frac{L_2 \cos \Psi}{n}$

$= (-\frac{1{,}729300753 \times 10^{13}}{7{,}267138378 \times 10^{11}}) \times 1 + (-\mathbf{13^\circ\ 17'0{,}72''})$

$= -\mathbf{10^\circ\ 30'45{,}5''}$.

$t_2 = -\frac{m}{n} \cos(M - N) - \frac{L_2 \cos \Psi}{n}$

$= (-\frac{1{,}729300753 \times 10^{13}}{7{,}267138378 \times 10^{11}}) \times 1 - (\mathbf{13^\circ\ 17'0{,}72''})$

$= -\mathbf{37^\circ\ 04'46{,}94''}$

[60] The value of quadrant $\cos \Psi$ is positive for the early phase of eclipse. See William Marshall Smart, *Textbook on Spherical Astronomy*, (Great Britain: University Press, Cambridge, 1977), 397-398.

[61] The value of quadrant $\cos \Psi$ is negative, for the end phase of eclipse. See William Marshall Smart, *Textbook on Spherical Astronomy*, (Great Britain: University Press, Cambridge, 1977), 398.

The value of (-$37^\circ\ 04'46,94''$), must be adjusted to the time value of one day which is $24^\circ\ 0'0''$.

Then t_1, $37^\circ\ 04'46{,}94'' - 24^\circ\ 0'0'' = 13^\circ\ 04'46{,}94''$.

So the beginning of time an eclipse is,

$\frac{m_1}{n_1}\cos(M_1 - N_1) - \tau_1 = T_1$

$10^\circ\ 30'45{,}5''$[62] $- (-13^\circ\ 17'0{,}72''$[63]$) = \mathbf{23^\circ 47'46,22''\ GMT}$.

As for the result of the end time of an eclipse,

$\frac{m_2}{n_2}\cos(M_2 - N_2) + \tau_2 = T_2$

$13^\circ\ 04'46{,}94''$[64] $+ (13^\circ\ 17'0{,}72''$[65]$) = \mathbf{26^\circ 21'47,66''\ GMT}$.

The value of $26^\circ 21'47,66''$, must be adjusted to the time value of one day which is $24^\circ\ 0'0''$.

Then T_2, $26^\circ 21'47{,}66'' - 24^\circ\ 0'0'' = \mathbf{2^\circ\ 21'47,66''}$ GMT.

So, from calculation above it is known that:

a. The beginning of time a Solar eclipse is $\mathbf{23^\circ 47'46,22''\ GMT}$ ($\mathbf{6^\circ 47'46,22''\ LMT}$)

b. The end of time a Solar eclipse is $\mathbf{2^\circ\ 21'47,66'}$ GMT ($\mathbf{9^\circ\ 21'47,66'}$ LMT)

The Results Comparison of Algorithm Solar Eclipse W.M. Smart with NASA

Time of Solar Eclipse

W.M. Smart Algorithm	NASA Database[66]
$\mathbf{6^\circ 47'46,22''\ LMT}$	$\mathbf{6^\circ 47'18''\ LMT}$

Therefore, the difference between them is just 4 seconds aparts.

D. Conclusion

A calculation of a solar eclipse by W.M. Smart uses a Besselian spherical function where the system emphasizes the coordinates of some heavenly bodies, namely the sun, earth, and

[62] Hasil dari perhitungan t_1 .

[63] Hasil dari perhitungan $\frac{L_2 \cos\Psi}{n}$ dengan konstanta $\cos\Psi$ positif.

[64] Hasil dari perhitungan t_2.

[65] Hasil dari perhitungan $\frac{L_2 \cos\Psi}{n}$ dengan konstanta $\cos\Psi$ negatif.

[66] Solar Eclipse Database by NASA, cam be seen at https://eclipse.gsfc.nasa.gov/SEpath/SEpath2001/SE2016Mar09Tpath.html.

moon. They calculated that a lot of basic Bessel calculations and trigonometry of spherical triangles applied astronomically made the calculations for a solar eclipse systematic. The results of a solar eclipse in the astronomy algorithm W.M. Smart have a difference that could be considered insignificant. Differences are only contained for several seconds in a time lapse. This statement has been demonstrated over time through a comparison of W.M. Smart's spherical astronomy algorithms and the NASA database algorithms, with time differences of just over four seconds.

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