Tracking the Acceleration Due to Gravity and Damping of a Pendulum: A Video Analysis

Frank Angelo Pacala[1*], Unofre Pili[2]

[1]Presidential School of Uzbekistan, Uzbekistan
[2]University of San Carlos, Philippines

ARTICLE INFO

ABSTRACT

This study focused on using video analysis to track the acceleration due to gravity and damping of a pendulum. The video tracker analysis software was proven effective in collecting data from several physics experiments. This study offered new insights on using a tracker and combining it with MS Excel to generate a damping graph. A pendulum was set in motion and recorded with a high-speed camera from the iPhone 13. The footage was then analyzed using tracker video analysis, modeling software, and Microsoft Excel to determine the amplitude and frequency of the pendulum's motion. At the same time, the other variables were measured using traditional tools (e.g., the length of the yarn using a meter ruler). The measured values were then used to calculate the acceleration due to gravity and the damping constant of the system. The results showed that the acceleration due to gravity was consistent with the average accepted value, while the damping constant accurately represented what was seen from the generated graph. Overall, this study demonstrated the effectiveness of video analysis in tracking the behavior of physical systems and its potential applications in various fields.

Introduction

The concept of acceleration due to gravity is one of the fundamental concepts in physics, and its importance in the classroom cannot be overstated. Gravity is one of the most fundamental forces in the universe, and the concept of acceleration due to gravity is critical to understanding how it works. Vidak, Šapić, and Mešić (2021) argued that by learning about gravity in the classroom, students can gain a better understanding of how objects interact with one another, both on Earth and in space. Understanding acceleration due to gravity is essential not just for physics, but for many other fields as well. For example, engineers designing roller coasters or spacecraft must understand how gravity affects their designs (Sohn & Hansen, 2019). Geologists studying the movement of

tectonic plates also need to understand the role of gravity in plate motion (Niu, 2018).

Moreover, damping in a pendulum is essential to understanding the behavior and stability of a pendulum's motion. Damping refers to reducing or eliminating the oscillations of a pendulum's movement over time (Kecik & Mitura, 2020). In real-world scenarios, some friction or resistance always opposes the movement of a pendulum. Without damping, a pendulum's oscillations would continue indefinitely, leading to instability and unpredictable behavior (Pfeiffer et al., 2023).

A few standard classroom experiments can be used to measure the acceleration due to gravity. A simple pendulum experiment can determine the acceleration due to gravity and damping. A pendulum's period is

related to its length and acceleration due to gravity. Therefore, by measuring the period of a pendulum, students can calculate the acceleration due to gravity. A damping experiment in the classroom can be conducted to demonstrate the effects of air resistance on a simple harmonic motion.

Measurement of the acceleration due to gravity and damping using advanced tools is not new in physics education. The study of Fatmala, Suyanto, Wahyudi, and Herlina (2020) studied the acceleration due to gravity together with their student using the Phyphox application, while Bura, Jufriansah, and Donuata (2022) used the Phyphox applications to improve learning outcomes, especially in the area of early knowledge acquisition and response including in the concept of acceleration due to gravity. The research conducted by Martin, Frisch, and Zwart (2020) measured the acceleration due to gravity using video analysis and yielded a 19.67 percent error. Fitria and Darma (2022) used a tracker to measure the damping constant of a spring in air and soapy water. This study is unique because most of the literature cited did not measure the damping constant or generate a graph of damping, which can show the students that there is a force of air resistance that causes the oscillation of the pendulum to damp. Also, this research measured the experiment's error propagation, which is not commonly found in other literature.

The choice of Tracker's Video Analysis Software over alternative applications is driven by its distinct combination of accuracy, portability, and user-friendliness to understand the acceleration of gravity and pendulum damping. Zhou et al (2019) argued that Tracker's advanced algorithms ensure precise tracking of frame by frame, a critical feature for accurately capturing the subtleties in pendulum motion. The software's flexibility, which enables it to accommodate different experimental settings and enhance its applicability for various physics studies, is beneficial (Pennington et al., 2019).

This study tracked the acceleration due to gravity and the damping constant of a simple pendulum attached to a wool rug yarn. The objectives of this paper are: 1) determine the relationship between period and length of cord in a simple pendulum; 2) calculate the acceleration due to gravity of a simple pendulum using tracker video analysis; and 3) compute the damping constant of a simple pendulum.

A pendulum is a simple mechanical device consisting of a weight called a bob suspended from a rod or string. When the pendulum is set into motion, it swings back and forth in a regular pattern. The motion of a pendulum is governed by several factors, including the length of the pendulum, the mass of the bob, and the acceleration due to gravity. The acceleration due to gravity is a fundamental physical constant that determines the strength of the gravitational force between two objects.

The acceleration due to gravity plays a crucial role in determining the speed and period of the pendulum's motion. The acceleration due to gravity can be considered the force that pulls the pendulum bob downward towards the earth's center. The faster the bob moves, the greater the gravitational force acting on it, resulting in increased acceleration.

To derive the period of a simple pendulum, the resultant force (F) on the bob equals Equation 1.

$$F = mg \sin \theta \qquad (1)$$

Figure 1
The Components of the Force of a Simple Pendulum (Rajasekaran, 2009)
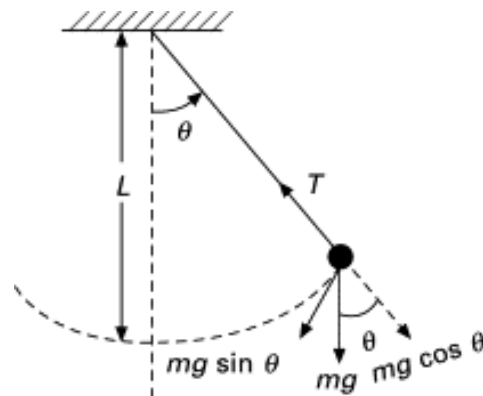
This force equals the general resultant force: F = mass (m) times acceleration (a). Equating equation 1 and F = m a yielded to Equation 2.

$$a = g \sin \theta \qquad (2)$$

Since the angle used in this experiment is small, a simple harmonic motion equation a = -angular frequency squared (ω^2) times displacement (x) can be utilized. The displacement x can be calculated using arclength (L) times the angle θ. Equating the simple harmonic motion equation and equation 2 and using x = Lθ resulted in Equation 3.

$$g \sin \theta = -(\omega^2) L\theta \qquad (3)$$

This experiment used a slight angle, so following the small angle principle, the sin θ = θ. The angular frequency equals 4π divided by the period (T). Finally, the period T would be Equation 4.

$$T = 2\pi \sqrt{\frac{L}{g}} \qquad (4)$$

Equation 4 tells us that the period is proportional to the square root of the length of the period square, which is directly related to the cord where the bob is connected. This means that if the length of a pendulum is increased or decreased, the pendulum's period will also increase or decrease respectively. Therefore, a longer pendulum will have a more extended period, while a shorter one will have a shorter one. This relationship was discovered by Galileo Galilei and is known as the law of isochronism (Macuglia, 2021).

If the gradient of the period squared versus length is plotted, the line formed has a gradient (G) equivalent to Equation 5.

$$G = \frac{g}{4\pi^2} \qquad (5)$$

Therefore, the value of the acceleration due to gravity (g) in Equation 6.

$$g = 4\pi^2 G \qquad (6)$$

The g has a value of approximately 9.8 ms^{-2} near the Earth's surface. This means that if a pendulum is dropped from a height of one meter above the ground, it will take approximately one second to reach the ground, and its speed will be approximately 9.8 ms^{-2}.

Moreover, when a pendulum swings back and forth, it experiences damping, which is a process that causes its amplitude to decrease over time. Damping is caused by air resistance or friction in the pendulum's pivot. The pendulum in this experiment experiences an under-damping or light damping. In light damping, the energy loss rate is relatively slower, and the characteristic damping time for the system is relatively longer compared to the heavily damped or critically damped system. This means a system with light damping will take longer to reach a steady-state condition than other systems with higher damping. According to MIT OpenCourseWare (2011), the general solution for the displacement of an underdamped oscillator is shown in Equation 7.

$$x(t) = Ae^{-bt/2m} \cos(\omega_d t - \varphi) \qquad (7)$$

In equation 7, the m is the mass, b is the damping constant, ω_d is the angular frequency, and t is the period. In a system with damping, the value of the damping constant affects the system's behavior. Specifically, higher values of the damping constant will cause the oscillations to decay more quickly, while lower values will result in slower decay. Therefore, the damping constant is critical in determining how quickly a system will reach equilibrium after being perturbed.

Methods

All materials of this experiment are found in the physics laboratory of the Presidential School of Uzbekistan, Qarshi City, Uzbekistan. This is also the location of the experiment. As seen in Figure 2, two 60 cm retort stands were assembled. These two stands faced each other by a distance of 1.5 m.

The mobile phone was clamped on the other stand with a view of where the oscillation happened. A 55g load was attached to the rug yarn and fixed on the c-clamped on the other retort stand. The load was oscillated at an angle of 15 degrees. This position was constant throughout the experiment. Six oscillations were performed with varying lengths of the yarn. Each of these oscillations was recorded in a high-speed camera of iPhone 13.

Figure 2
*The Materials in This Experiment*
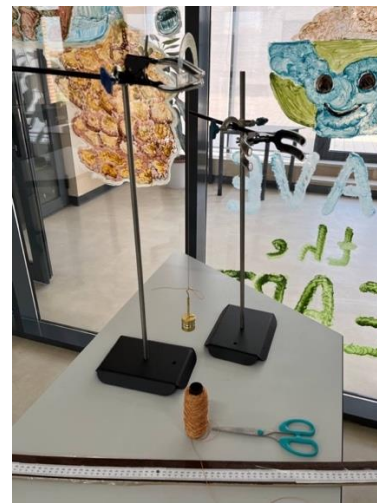
The video was uploaded to the tracker. The video was filtered by rotating it depending on the alignment of the video; in this experiment, it was rotated to 180 degrees. A calibration stick was inputted using the length of the boss-clamped holder. In this paper, the boss was 115 cm in length. Additionally, the coordinate axis was shifted towards the mass or load. This coordinate axis is shown as the purple crossed line, while the calibration stick is the blue line parallel to the boss, as shown in Figure 3.

The button tracked was clicked, and the center of mass and auto track. To start the auto tracking, the key used was the shift plus command in MacBook. The auto

tracker was paused when the needed data were collected.

Figure 3
The Interface of the Tracker Video Analysis and Modelling Software. On The Right Side is The X-Displacement Versus Time Generated by the Software
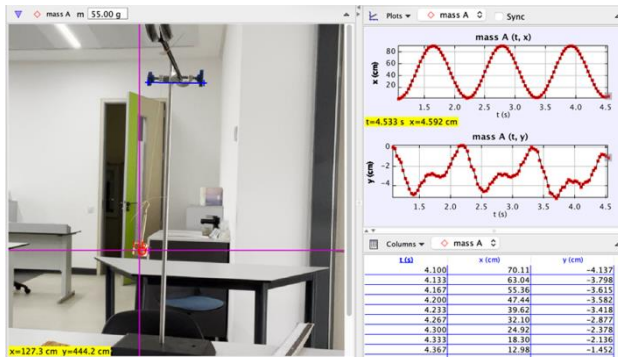
Experimental Procedure

The independent variable of this experiment is the length of the yarn used, while the dependent variable is the period. This is easily spotted in Equation 4. The length of the yarn started from 33.2 cm to 10.5 cm and was measured using a meter ruler. There were six lengths in total. The period was measured by taking the difference of time from one amplitude to the next amplitude using the graph of x-displacement versus time generated in the tracker.

The variables that remained constant were the mass of the load, the air resistance inside the physics laboratory, the starting angle of oscillation, and the type of rug yarn used. The mass of the load was measured using a mass balance of 55g. The angle of oscillation was taken by utilizing a protractor. A length versus period squared graph was plotted using Microsoft Excel when all data were gathered. A linear best fit indicated a direct relationship between length and time squared. The gradient of this line is Equation 5, and acceleration due to gravity is calculated using Equation 6.

The damping constant is measured using equation 7. The value of the spring constant of the rug yarn used was calculated by dividing the load's weight by the yarn's extension. The extension of the yarn was 1mm by a load of 400g so the spring constant of the yarn is an average of 3924 Nm^{-1}.

Result and Discussions

Acceleration due to Gravity

The variables needed to find the acceleration due to gravity are the period and length of the rug yarn. Figure

4 is a sample of how the period can be measured. The data points came from the tracker and were plotted in Microsoft Excel. To measure the period from a displacement-time graph of a pendulum, the one complete oscillation of the pendulum on the graph was identified. Then, the time it takes for the pendulum to complete one full oscillation was measured by calculating the time interval between two consecutive points on the graph where the pendulum is at the same position and moving in the same direction.

Figure 4
The Period of The Simple Pendulum, Whose Length Is 10.5 cm is 0.97 Seconds
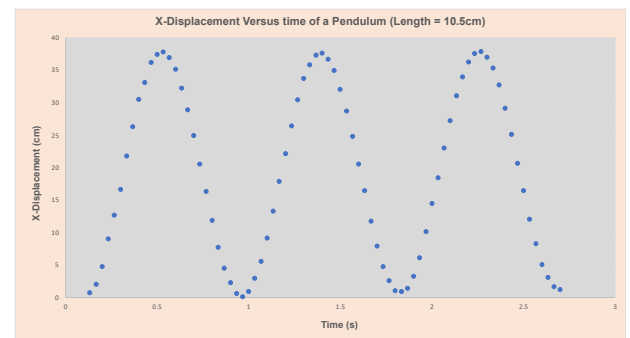
The distance between these two points on the graph represents the time for one complete oscillation, equivalent to the pendulum's period. So, Figure 4 is the period of amplitude two minus the period of amplitude, equal to 0.97 seconds. This same process was used to measure the period of the other lengths.

Equation 4 predicted that the trend for the data of length and period must be increasing. According to this equation, as the pendulum's length increases, the period also increases, but not proportionally. This relationship is nonlinear, and the period increases more slowly than the pendulum's length. This is shown in table 1. However, on period squared versus time, we can see that the period squared is directly proportional to the pendulum's length. As the pendulum length increases, the period squared will also increase. This relationship is known as a direct proportion.

Table 1
The Period of Varying Lengths of the Yarn in a Pendulum

Length (cm)	Period (s)	Period Squared (s²)
33.2	1.29	1.67
29.5	1.21	1.46
24.5	1.13	1.28
20.0	1.05	1.1
15.5	0.97	0.94
10.5	0.87	0.76

In Figure 5, the data in Table 1 were plotted using Microsoft Excel. As observed from the plot, the length of the wire of a pendulum versus the period squared, a linear trend with a positive slope can be observed. The slope should be equal to $g/4\pi^2$ or the equation 5, where g is the local acceleration due to gravity. A straight line of best fit is formed, which proves that the theory in equation 4 is correct. This means that the experiment has taken the correct data. Figure 5 presents a line of best fit that has a gradient of 25.431. This gradient was substituted in equation 6 to compute the acceleration due to gravity. The g was 1003 cm/s² or 10.03 ms⁻². This 10.03 ms⁻² computed g is near the average value of g, which is 9.81 ms⁻². Using the percent error formula, the percentage error on this experimental value of g is only 2.24%.

This percent error is lower than the experiment of Fatmala, Suyanto, Wahyudi, and Herlina (2020); however, the authors used the phyphox mobile application. Also, the study of Martin, Frisch, and Zwart (2020) came up with a value of g, which is 11.74 ms-2, and this has a percent error of 19.7%. The procedure of this experiment is better than Martin, Frisch, and Zwart (2020) because both used video analysis as a tool for finding the value of g. Wabeto (2022) calculated g with a value of 9.79 ms⁻² with a percent error of 0.20%; however, he used different tools like Adobe Photoshop and MATLAB programming algorithms to track the simple pendulum's movement.

Figure 5
The Gradient of the Length Versus Period Squared of a Simple Pendulum Is 25.431 cms⁻²
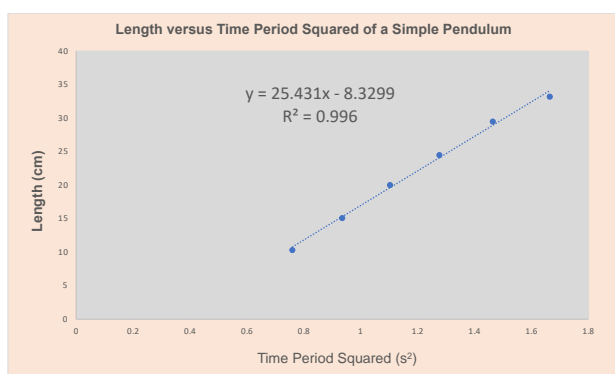
The acceleration uncertainty due to gravity can be calculated using the error propagation formula on division and power, as shown in Equation 8.

$$\Delta g = g\left(\frac{\Delta L}{L} + 2\frac{\Delta T}{T}\right) \qquad (8)$$

The uncertainty on the length is based on the minor division of the ruler, 1mm or 0.1cm (Fitzgerald et al., 2011). The uncertainty on the tracker is based on the literature. According to Marzari, Rosi, and Onorato (2021), the uncertainty of the tracker is ±0.1 ms⁻². Martin, Frisch, and Zwart (2020) also mentioned that the uncertainty is ±0.3 ms⁻², and Anni (2021) reported a ±0.08 ms⁻² uncertainty. By taking an average of the uncertainty based on this literature, the uncertainty used by this experiment on the period is 0.3s. Finally, this experiment reports that the acceleration due to gravity computed is (10.03 ± 0.3) ms⁻². The accepted value of g in Uzbekistan is 9.80 ms⁻², as determined by an online gravitational acceleration calculator. This calculator requires the user to enter the experiment's location's latitude and elevation.

The lower limit of the value of g in this experiment is 9.73 ms⁻², which is shorter than the theoretically acceptable value in the country. The error attributed to this is that the air resistance affects the experiment. The tracker is an ultra-sensitive device, so the placement of the calibration stick should be appropriately positioned; this experiment may have positioned the calibration stick a little off from the clamped boss.

Damping and its Constant

Damping in a pendulum is caused by energy dissipation due to frictional forces such as air resistance, the bearing friction at the pivot, and the internal friction within the pendulum itself (Xiang, 2023). As the pendulum swings back and forth, it experiences resistance from the surrounding air molecules, which causes a loss of energy, resulting in the dampening of its motion over time. As a result, the amplitude of the pendulum's oscillations decreases with time until it comes to rest.

Figure 6
The Pendulum of Length 15.5 cm Underwent a Light Damping on Air. The Damping Constant Is 1.38 kgs⁻¹
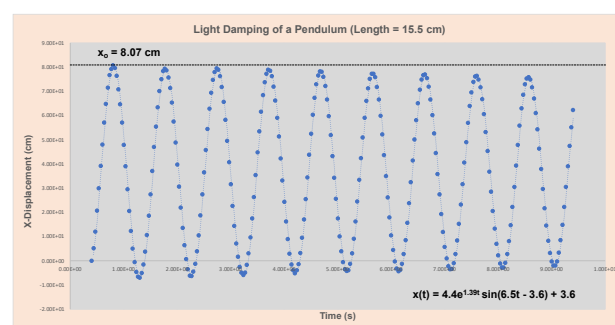
As observed from Figure 6 and Figure 7, the pendulum exhibited damped oscillations where the oscillation amplitude decreases over time but may do so with a slow decay rate. The maximum displacement in Figure 6 is 8.07 cm and decays exponentially. This type of damping is called underdamped or light damping. Underdamping in a pendulum refers to a situation where the pendulum is subjected to a damping force that is less than the critical damping required to prevent oscillations from continuing indefinitely (Tahira et al., 2020).

The damping equation in Figures 6 and 7 follows a sine wave. The tracker video analysis software can determine the sine wave equation of this damping by going into the analysis section of the generated graph and choosing a damped sine as the analysis tool. The equation in the tracker is similar to equation 7.

Based on Equation 7, the damping constant is represented by b. The damping constant in Figure 6 is 1.38 kgs^{-1}, and Figure 7 is 1.1 kgs^{-1}. The damping constant in a pendulum represents the amount of resistance to the motion of the pendulum due to external factors such as air resistance or friction (Hafez, 2022). A higher damping constant means more damping and a quicker reduction in amplitude, while a lower damping constant results in a slower amplitude reduction. This only means that the pendulum in Figure 6 has a faster mass per time in settling to the resting position.

Figure 7

The Pendulum Of Length 15.5 cm Underwent a Light Damping on Air. The Damping Constant Is 1.1 kgs-1.
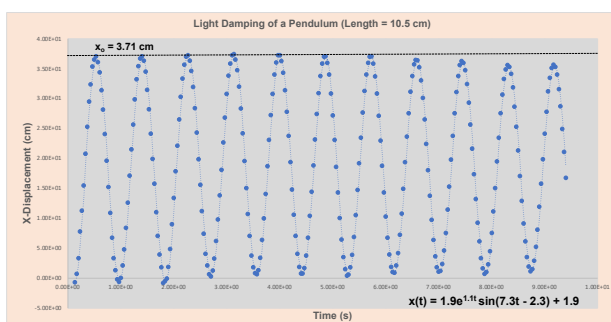
Implications

The results of this research have many implications, most especially in teaching and learning. Dynamic and exciting learning experiences are offered to students through modern educational technologies, such as video analysis and modeling software. This way of thinking may make physics more accessible and

exciting, which can foster an increased interest in the subject. This approach could make physics more accessible and exciting, giving rise to a broader interest in the subject.

This software allows students to apply theoretical physics concepts directly in real-world scenarios. Tracking the acceleration due to gravity and damping of a pendulum through video analysis bridges the gap between abstract theory and tangible, observable phenomena, enhancing students' understanding. The study provides an opportunity for hands-on experimentation and enables students to participate actively in scientific activities. Students can carry out experiments, collect data, and analyze results, supporting a more vigorous and experiential learning approach for physics.

Students acquire valuable quantitative analysis skills by measuring the pendulum's frequency, period, and deflection. Students will gain valuable quantitative analysis skills by measuring a pendulum's frequency, period, and deflection. Tracking and analyzing pendulum variables requires critical thinking and problem-solving skills. To encourage thinking of inquiry and analytical thought, students are asked to interpret the data, analyze its characteristics, and draw conclusions. To foster a comprehensive understanding of the topic, students can evaluate how physics concepts are used in other areas like engineering or biomechanics.

The success of this application may lead to advancements in educational technology tailored for physics education. Innovations in software and hardware could further enhance the accuracy and ease of conducting experiments, making physics more accessible to a broader range of students. This approach can enable teachers to produce vivid learning experiences through visual metaphors that enhance the effectiveness of teaching complex concepts clearly and comprehensibly.

Conclusions

Based on the analysis of the video, the acceleration due to gravity and damping of a pendulum can be accurately tracked and calculated. Through the use of various techniques such as measuring the amplitude and period of the pendulum's oscillation, and analyzing the video with tracker video analysis and modeling software, precise and near-accurate measurements were obtained. The experiment highly

depended on modern educational technologies such as mobile phones and computers. The acceleration due to gravity was (10.03 ± 0.3) ms^{-2}; this value's lower limit is 9.73 ms^{-2}. This value is close to the accepted average value of g because the percent error was only 2.24%. In addition, the tracker has also tracked the damping of the pendulum in air and calculated the value of the damping constant. Moreover, the data that were calculated and gathered can offer valuable insights into the behavior of a pendulum and its interactions with its environment. Accurately tracking these variables can also aid in designing and developing pendulum-based systems used in various fields, ranging from engineering to physics. Furthermore, tracking the acceleration due to gravity and damping of a pendulum through video analysis allows students and researchers to apply their knowledge of physics concepts to real-world scenarios. It can also serve as an effective tool for demonstrating scientific principles in a visually compelling way.

References

Anni, M. (2021). Quantitative Comparison between the Smartphone Based Experiments for the Gravity Acceleration Measurement at Home. *Education Sciences*, *11*(9), 493.

Bura, Y., Jufriansah, A., & Donuata, P. B. (2022). The Effect of Using Phyphox Applications to Improve Learning Outcomes Reviewed from Early Knowledge and Response. *Jurnal Pendidikan Fisika dan Teknologi*, *8*(2), 203-207.

Fatmala, F. D., Suyanto, E., Wahyudi, I., & Herlina, K. (2020). The Effect of GIL Assisted Phyphox in Physics Learning towards Creative Thinking. *Jurnal Pembelajaran Fisika*, *8*(2), 148.

Fitria, N.S.I & Darma, N.M. (2022). Pengembangan Modul Praktikum Osilasi Teredam Pada Pegas Berbasis Analisis Video Berbantuan Tracker. *UPEJ Unnes Physics Education Journal*, *12*(2), 9-16.

Fitzgerald, M. T., McKinnon, D., Danaia, L., & Woodward, S. (2011). Using smartphone camera technology to explore stellar parallax: Method, results, and reactions. *Astronomy Education Review*, *10*(1), 1-8.

Hafez, Y. (2022). Solving the nonlinear pendulum equation with friction and drag forces using the Finite Element Method. *The Romanian Journal of Technical Sciences. Applied Mechanics.*, *67*(2), 137-161.

Kanga, L. K., Astro, R. B., & Ika, Y. E. (2021). Analysis of Vertical Movements on Freedom and Bullet Movement Events using Video Tracking Method. *ScienceEdu: Jurnal Pendidikan IPA*, *4*(1), 19-25.

Kecik, K., & Mitura, A. (2020). Energy recovery from a pendulum tuned mass damper with two independent harvesting sources. *International Journal of Mechanical Sciences*, *174*, 105568.

Macuglia, D. (2021). The universe: A book written in the mathematical—and the programming—language. *Il nuovo cimento C*, *44*(1), 1-13.

Martin, T., Frisch, K., & Zwart, J. (2020). Systematic Errors in Video Analysis. *The Physics Teacher, 58* (3), 195. https://doi.org/10.1119/1.5145415

Marzari, A., Rosi, T., & Onorato, P. (2021). Three experiments on the Einstein's lift. *European Journal of Physics*, *42*(4), 045404.

MIT OpenCourseWare (2011). Under, Over and Critical Damping. Retrieved from https://ocw.mit.edu/courses/18-03sc-differential-equations-fall-2011/7e212064ad281d00e1dac893b1f722a7_MIT18_03SCF11_s13_2text.pdf

Niu, Y. L. (2018). Geological understanding of plate tectonics: basic concepts, illustrations, examples and new perspectives. *Global tectonics and metallogeny.*, *10*(1), 23-46.

Pennington, Z. T., Dong, Z., Feng, Y., Vetere, L. M., Page-Harley, L., Shuman, T., & Cai, D. J. (2019). ezTrack: An open-source video analysis pipeline for the investigation of animal behavior. *Scientific reports*, *9*(1), 19979.

Pfeiffer, K., Escande, A., Gergondet, P., & Kheddar, A. (2023). The Hierarchical Newton's Method for Numerically Stable Prioritized Dynamic Control. *IEEE Transactions on Control Systems Technology*, 1-14.

Rajasekaran, S. (2009). *2 - Free vibration of single-degree-of-freedom systems (undamped) in relation to structural dynamics during earthquakes.* Woodhead Publishing Series in Civil and Structural Engineering, Bangkok, Thailand.

Sohn, E., & Hansen, A. (2019). *Models and Designs.* Norwood House Press.

Tahira, A. H., Ramzan, M., & Khalid, M. (2020). Impulsive Response of Damped Harmonic Oscillator Via Fourier Transform. *Nature and Science*, *18*, 12.

Vidak, A., Šapić, I. M., & Mešić, V. (2021). An augmented reality approach to learning about the force of gravity. *Physics Education*, *56*(6), 065026.

Wabeto, A. (2022). *Determining of acceleration due to gravity using video image processing.* Available at SSRN: https://ssrn.com/abstract=4066133 or http://dx.doi.org/10.2139/ssrn.4066133

Xiang, Y., Tan, P., He, H., Yao, H., Zheng, X., & Yang, K. (2023). A novel bi-directional rail variable friction pendulum-tuned mass damper (BRVFP-TMD). *Mechanical Systems and Signal Processing*, *197*, 110396.

Zhou, S., Ke, M., Qiu, J., & Wang, J. (2019). A survey of multi-object video tracking algorithms. In *International Conference on Applications and Techniques in Cyber Security and Intelligence ATCI 2018: Applications and Techniques in Cyber Security and Intelligence* (pp. 351-369). Springer International Publishing.