e-ISSN: 2714-5506
p-ISSN: 2714-609X

SQUARE : Journal of Mathematics and Mathematics Education
Volume 4, No. 2, 2022, pp. 117-123
http://dx.doi.org/10.21580/square.2022.4.2.13212

Annual Premium Determination for Joint Life Insurance with De Moivre and Gompertz's Mortality Laws

Seftina Diyah Miasary
UIN Walisongo Semarang
dseftina@yahoo.co.id

ABSTRAK

Penelitian ini membahas tentang cara menentukan premi tahunan untuk status hidup bersama. Tingkat kematian (*mortality rate*) merupakan salah satu faktor yang dapat dipertimbangkan dalam perhitungan premi. Metode kuantitatif yang digunakan dalam penelitian ini adalah mengestimasi parameter hukum Mortalitas De Moivre dan Gompertz dengan menggunakan data sekunder dari Tabel Mortalitas Indonesia (TMI III) tahun 2011 untuk pria dan wanita. Perhitungan ini menghasilkan rumus untuk menghitung premi tahunan asuransi jiwa bersama berdasarkan hukum kematian De Moivre dan Gompertz. Dimulai dengan memperkirakan parameter hukum kematian Gompertz untuk Tabel Kematian Indonesia tahun 2011 dengan menggunakan metode estimasi kemungkinan maksimum, kemudian menghitung probabilitas hidup gabungan, APV manfaat kematian, APV anuitas hidup berkelanjutan, dan premi tahunan untuk asuransi jiwa bersama. Nilai premi tahunan asuransi jiwa gabungan dengan hukum kematian De Moivre dan Gompertz untuk simulasi asuransi jiwa berjangka n = 10 tahun dengan umur x (suami) 28 tahun dan y (istri) 25 tahun, manfaat kematian (R) adalah Rp. 50.000.000; dan tingkat bunga 3,50 persen dengan Tabel Mortalitas Indonesia tahun 2011. Menurut perhitungan, nilai premi tahunan asuransi jiwa gabungan berdasarkan hukum mortalitas Gompertz lebih besar daripada hukum mortalitas De Moivre.
Kata Kunci: *join life insurance*, hukum kematian, premi tahunan.

ABSTRACT

This research discusses how to determine the annual premium for joint life status. The death rate (mortality rate) is one of the factors that can be considered when calculating the premium. The quantitative method used in this study was to estimate the parameters of the De Moivre and Gompertz's Mortality laws using secondary data from the 2011 Indonesian Mortality Table (TMI III) for men and women. This calculation generates a formula for calculating the annual premium for joint life insurance based on De Moivre and Gompertz mortality law. Starting with estimating the parameters of Gompertz's mortality law for the Indonesian Mortality Table in 2011 using the maximum likelihood estimation method, then calculating the combined life probability, death benefit APV, continuous life annuity APV, and annual premium for joint life insurance. The value of the annual premium on joint life insurance with the mortality law of De Moivre and Gompertz for a simulated term life insurance n = 10 years with age x (husband) 28 years and y (wife) 25 years, the death benefit (R) is Rp. 50,000,000; and the interest rate is 3.50 percent with the Indonesian Mortality Table in 2011. According to the calculations, the annual premium value of joint life insurance based on Gompertz's mortality law is greater than De Moivre's mortality law.
Keywords: *Join Life Insurance, Mortality Laws, Annual Premium.*

1. INTRODUCTION

The insurance industry is a non-bank financial institution that plays an important role in society, one of which is collecting and distributing funds generated by insurance premiums (Andriyani, 2017). The premium is the amount of money that the insured must pay every month in order to participate in insurance. The insurance company determined the size of the premium that must be paid by adjusting the insured's condition. The size of the required premium has been determined by the insurance company by adjusting the insured's condition (Lestari et al., 2019). There are several types of premiums, one of which is an annual premium. This premium is paid at the start of each year, and the amount can vary or remain the same year after year. The annual premium is determined by calculating the Actuarial Present Value (APV) of the death benefit value and the APV of the life annuity, which are influenced by interest rates, life chances, and mortality opportunities in a given period (Harisa et al, 2013).

The type of insurance used also influences the premium calculation. In Indonesia, there are two types of life insurance: combined life insurance (multiple life) and individual life insurance (single life) (Putri, 2019). In this study, the annual premium in joint life insurance is analyzed using the law of mortality to provide an overview of a population's death rate via a mortality table. The mortality laws of De Moivre and Gompertz can be used to describe the death rate in a population (Sesilia, 2017). De Moivre's and Gompertz's laws of mortality have some similarities, such as discussing a population's mortality rate, but they also differ. The form of life probability in De Moivre's law of mortality is linear, whereas the form of life probability in Gompertz's law of mortality is exponential (Mitus, 2016). Based on these facts, this study will compare the determination of annual premiums for joint life life insurance using De Moivre and Gompertz's mortality law.

2. METHOD

This study uses simulation to calculate the annual premium using De Moivre and Gompertz's mortality law. In this simulation, the insurance calculated is life insurance with a term of n = 10 years, the age of signing the contract when y (wife) = 25 years and x (husband) = 28 years. The benefit (R) is assumed to be IDR 50,000,000, and the mortality table used as a reference for calculating premiums is TMI III in 2011 for women and men, where TMI III 2011 is used to determine premium prices in the Indonesian life insurance industry. Furthermore, the Interest rate i is based on the 3.5 percent Indonesian bank interest rate for November 2021.

The steps for calculating the annual premium for joint life insurance using De Moivre and Gomperz's mortality law are as follows:

1. Calculating continuous interest rates (δ)

   The continuous interest rate is an important factor in calculating the annual premium (Bhuana, dkk., 2015). The continuous interest rate is expressed as a percentage of the value at time t as the derivative of the accumulated value over time. A continuous interest rate with a constant value over a given time interval will produce a constant effective interest rate (i) as shown below (Rakhman and Effendie, 2013),

$$\delta = \ln(1+i) \qquad (1)$$

2. Estimating the survival function parameters from De Moivre and Gompertz's laws
   Mortalita law is an approach used to postulate an analytic form of mortality or life chances (Manjaruni dan Purnaba, 2021). The law of mortality that will be used in this research is the law of mortality of De Moivre and Gompertz. The life probability function of a person aged x to t years according to De Moivre's law of mortality is (Bowers et al, 1997)

$$ {}_tp_x = \frac{\omega - x - t}{\omega - x} \qquad (2) $$

   Where ω is the maximum age for obtaining insurance. The survival function of a person aged x to t is then, according to Gompertz's law,

$$ {}_tp_x = \exp\left[-\frac{Bc^x}{\log c}\left(c^t - 1\right) \right] \qquad (3) $$

   Where B and c are the parameters of Gompertz's law of mortality. Furthermore, parameters B and c are estimated using the Maximum Likelihood Estimator (MLE).

3. Calculating the probability of join life
   If (x) and (y) are the ages of two people when they take out insurance, where $T(x)$ and $T(y)$ represent the remaining age variables for x and y, respectively. Furthermore, $T(xy)$ is the variable that will live until one of x or y dies, as denoted by $T(xy) = \min\{T(x), T(y)\}$. The joint life status probability function can then be written as follows (Sertdemir, 2013),

$$ F_{T(xy)}(t) = {}_tq_{xy} = {}_tq_x + {}_tq_y - {}_tq_x \cdot {}_tq_y \qquad (4) $$

   Where ${}_tq_x$ and ${}_tq_y$ represent the probability that someone aged (x) or (y) dies before reaching the age of (x) and $(x+t)$ years. Meanwhile, the survival function for the joint life case can be written as the following equation:

$$ S_{T(xy)}(t) = 1 - F_{T(xy)}(t) \qquad (5) $$

4. Calculating the Actuarial Present Value (APV) of the death benefit from the n-year term joint life insurance
   Bowers et al (1997) states that the APV value for n-year term life insurance with a death benefit is paid after the insured's death. Under De Moivre's law of mortality, the APV value of the death benefit can be written as follows:

$$ \bar{A}^1_{x:\overline{n}|} = \int_0^n e^{-\delta t}\left(\frac{\omega - x - t}{\omega - x}\right).\left(\frac{\omega - y - t}{\omega - y}\right) \left[\left(\frac{1}{\omega - x - t}\right).\left(\frac{1}{\omega - y - t}\right)\right]\left[\mu(x+t).\mu(y+t)\right]dt \qquad (6) $$

   Where $\bar{A}^1_{x:\overline{n}|}$ is the APV value of the n-year term life insurance. Meanwhile, Gomperz's law of mortality is as follows (Bowers et al, 1997),

$$ \bar{A}^1_{x:\overline{n}|} = \int_0^n e^{-\delta t} e^{\left[-\frac{Bc^x}{\log c}\left(c^t - 1\right)\right]}.e^{\left[-\frac{Bc^y}{\log c}\left(c^t - 1\right)\right]} \left[\left(Bc^{x+t}\right) + \left(Bc^{y+t}\right)\right]\left[\mu(x+t).\mu(y+t)\right]dt \qquad (7) $$

5. Calculating the Actuarial Present Value (APV) of the n-year term joint life annuity
   Life annuity is a series of payments made continuously as long as a person follows insurance (Dickson, Hardy, and Waters 2019). The APV of the annuity for De Moivre's law of mortality is,

$$\bar{a}^{1}_{xy:\overline{n}|} = \int_0^n e^{-\delta t}\left(\frac{\omega - x - t}{\omega - x}\right).\left(\frac{\omega - y - t}{\omega - y}\right) dt \qquad (8)$$

   Where $\bar{a}_{x:\overline{n}|}$ is the value of the n-year term annuity. Furthermore, the APV value of the n-year term annuity is based on Gompertz's law of mortality,

$$\bar{a}^{1}_{xy:\overline{n}|} = \int_0^n e^{-\delta t} e^{\left[-\frac{Bc^{x}}{\log c}\left(c^{t}-1\right)\right]}.e^{\left[-\frac{Bc^{y}}{\log c}\left(c^{t}-1\right)\right]} dt \qquad (9)$$

6. Calculating the annual premium of n-year joint life term insurance based on De Moivre and Gompertz's laws of mortality.
   The notation $\bar{P}\left(\bar{A}^{1}_{x:\overline{n}|}\right)$ denotes a continuous annual premium which can be determined by the equivalence principle. In general, the equation that can be used to determine the annual premium for n-year term insurance is as follows (Bowers et al, 1997)

$$\bar{P}\left(\bar{A}^{1}_{xy:\overline{n}|}\right) = \frac{\bar{A}^{1}_{xy:\overline{n}|}}{\bar{a}_{xy:\overline{n}|}} \qquad (10)$$

3. RESULT AND DISCUSSIONS

3.1. Calculating Continuous Interest Rates (δ)

The interest rate used in this study is 3.50 percent following the interest rate at Bank Indonesia. Furthermore, to calculate the annual premium for n-year term life insurance with joint life status, the death benefit is paid immediately after death is the continuous interest rate (force of mortality). By using equation (1), the continuous interest rate value is 0.0344014267173.

3.2. Estimating The Survival Function Parameters from De Moivre and Gompetz's Mortality Laws

The next step in calculating the annual premium for n-year joint life insurance is to estimate the De Moivre and Gompetrz mortality model parameters. The parameter in the De Moivre model represents the maximum age at which a person can obtain insurance. The maximum age for a person to obtain insurance (ω) is assumed to be 111 years in this study. Furthermore, the estimated parameters of the Gompertz model using MLE are B = 0.005749 and c = 1.024738.

3.3. Calculating the Joint Life Probability

The probability of surviving a single life at the age of x years to the age of (x+t) years is represented by Equations (2) and (3). Furthermore, the equation is modified for the case of multiple life, in which two insured people aged x and y years are used. Equations (2) and (3) are transformed as follows:

$$ {}_t p_{xy} = \left(\frac{111 - x - t}{111 - x} \right) \cdot \left(\frac{111 - y - t}{111 - y} \right) \qquad (11) $$

Equation (11) represents the survival function for a person aged x to t years according to De Moivre's mortality law, then the survival function for a person aged x to t years in accordance with Gompertz's law is

Table 1. APV of Death Benefit, APV of Annuity and Annual Premiums of Joint Life Term Insurance Based on De Moivre and Gompertz Mortality Law

n	$\overline{A}^{1}_{x:\overline{n}\|}$		$\overline{a}^{1}_{xy:\overline{n}\|}$		$\overline{P}\left(\overline{A}^{1}_{xy:\overline{n}\|}\right)$	
	De Moivre	**Gompertz**	**De Moivre**	**Gompertz**	**De Moivre**	**Gompertz**
5	0.009378	0.007990181	0.372624437	0.32161634	1.258.309,60	1.242.191,36
10	0.014798	0.011215595	0.551006105	0.39952099	1.342.826,25	1.403.630,25
15	0.017435	0.011349526	0.60558726	0.35779214	1.439.519,47	1.586.050,15
20	0.018165	0.009762866	0.585505449	0.27237435	1.551.226,55	1.792.177,91
25	0.017636	0.007485542	0.524339468	0.18481956	1.681.741,09	2.025.094,68
30	0.016322	0.005204345	0.444424624	0.11371729	1.836.253,37	2.288.282,01
35	0.014562	0.003298225	0.360073095	0.06377882	2.022.058,83	2.585.673,92
40	0.012597	0.001903738	0.279959844	0.03257911	2.249.747,21	2.921.716,14
45	0.010591	0.000996211	0.208875587	0.01508757	2.535.301,67	3.301.430,35
50	0.008654	0.000469150	0.149002108	0.00628804	2.904.040,40	3.730.496,38

$$ {}_t p_{xy} = \exp\left[-\frac{0.004749\left(1.024738\right)^{x}}{\log\left(1.024738\right)}\left(1.024738^{t} - 1\right) \right] \qquad (12) $$
$$ \exp\left[-\frac{0.004749\left(1.024738\right)^{y}}{\log\left(1.024738\right)}\left(1.024738^{t} - 1\right) \right] $$

Furthermore, using a simulation in which x = 28 years (husband's age), y = 25 years (wife's age), the agreed insurance period is n = 10 years, and the maximum age to take insurance (δ) is 111 years, the chances of survival are calculated. combined with the joint life status of the De Moivre model of 0.77724853 and the combined life probability of the joint life status of the Gompertz model of 0.56356382.

3.4. Calculatin Actuarial Present Value (APV) of Death Benefit From n-Year Term Joint Life Insurance

The APV of death benefit from n-year term joint life insurance can be calculated using equations (6) and (7). By using simulation data where x = 28 years, x = 25 years and n = 10 years, the APV value of the death benefit of the De Moivre's model of joint life insurance is

0.0147981092. As for the Gompertz's model, the APV value of the death benefit from joint life insurance is 0.0112155950.

3.5. Calculating the Actuarial Present Value (APV) of an n-Year Term Joint Life Annuity

The APV of n-year joint life annuities can be calculated using equations (8) and (9). Based on the simulation data with x = 28 years, x = 25 years and n = 10 years, the APV for the joint life annuity of the De Moivre model is 0.551006105 and the APV of the joint life annuity of the Gompertz model is 0.39952099.

3.6. Calculating the Annual Premium of n-Year Joint Life Term Insurance Based on De Moivre and Gompertz' Laws of Mortality

The annual premium from the n-year term joint life insurance can be obtained by dividing the APV of the death benefit from the n-year joint life life insurance by the APV of the n-year joint life annuity according to equation (10). The results of the calculation of the annual premium of De Moivre's mortality law model for x = 28 years, x = 25 years and n = 10 years are 1,342,826.2437 while the annual premium for the Gompertz model is 1,403,630.2347.

4. RESULT

The actuarial present values for the benefits and annuities of joint life term insurance produce different values over time. The resulting annual premium, on the other hand, will rise over time. According to the calculations, the annual premium of term joint life life insurance calculated using Gompertz's mortality law is greater than the annual premium calculated using De Moivre's mortality law.

5. REFERENCES

Bhuana, Tri Yana, I Nyoman Widana, And L U H Putu I D A Harini. 2015. "Menentukan Premi Tahunan Untuk Tiga Orang Pada Asuransi Jiwa Hidup Gabungan (Joint Life)." *E-Jurnal Matematika* 4(4): 195–200.

Bowers, N L et al. "Actuarial Mathematics, 1997." *Society of Actuaries*.

Dickson, David C M, Mary R Hardy, and Howard R Waters. 2019. *Actuarial Mathematics for Life Contingent Risks*. Cambridge University Press.

Harisa, Nurma, and Johannes Kho. 2013. "Premi Tahunan Asuransi Jiwa Berjangka Dengan Hukum De Moivre Untuk Status Gabungan."

Lestari, Dwi Ayu, Neva Satyahadewi, and Hendra Perdana. 2019. "Penentuan Cadangan Premi Asuransi Jiwa Dwiguna Berjangka Dengan Metode Illinois." *Bimaster: Buletin Ilmiah Matematika, Statistika dan Terapannya* 8(3).

Manjaruni, Vivin, and I Gusti Putu Purnaba. 2021. "Menentukan Premi Asuransi Jiwa Joint Life Untuk Tiga Orang Tertanggung." *Statmat: Jurnal Statistika Dan Matematika* 3(1): 29–38.

Mitus, Aswin. 2016. "Analisis Perbandingan Survival Function Dengan Hukum De Moivre Dan Hukum Gompertz."

Putri, Stefi Amalia. 2019. "Penentuan Premi Dari Asuransi Kontingensi (Contingent Insurance)." *Jurnal Matematika UNAND* 4(2): 81–88.

Rakhman, A, and A R Effendie. 2013. "Matematika Aktuaria." *Universitas Terbuka. Tanggerang Selatan*.

SE, Mellisa Fitri Andriyani M. 2017. "Penerimaan Premi Dan Pembayaran Klaim Di PT. Taspen (Persero) Cabang Yogyakarta."

Sertdemir, Beste Hamiye. 2013. "Multiple Life Insurance."

Sesilia, Susanti. 2017. "Premi Tahunan Asuransi Jiwa Seumur Hidup Dengan Hukum De Moivre."